

Quantum telepathy? Predicting your opponent's moves in the Bell game, and the Delft Bell experiment

Richard Gill
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Rutherford: If you need statistics, you did the wrong experiment

Hensen et al. (2015, Nature) Loophole-free Bell inequality violation
using electron spins separated by 1.3 kilometres

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with $N = 245$ and at significance level $p = 0.039$
They need sophisticated statistics and probability theory

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LETTER

doi:10.1038/nature15759

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen^{1,2}, H. Bernien^{1,2,†}, A. E. Dréau^{1,2}, A. Reiserer^{1,2}, N. Kalb^{1,2}, M. S. Blok^{1,2}, J. Ruitenbergh^{1,2}, R. F. L. Vermeulen^{1,2}, R. N. Schouten^{1,2}, C. Abellán³, W. Amaya³, V. Pruneri^{3,4}, M. W. Mitchell^{3,4}, M. Markham⁵, D. J. Twitchen⁵, D. Elkouss¹, S. Wehner¹, T. H. Taminiau^{1,2} & R. Hanson^{1,2}

More than 50 years ago¹, John Bell proved that no theory of nature that obeys locality and realism² can reproduce all the predictions of quantum theory: in any local-realist theory, the correlations between outcomes of measurements on distant particles satisfy an inequality that can be violated if the particles are entangled. Numerous Bell inequality tests have been reported^{3–13}; however, all experiments reported so far required additional assumptions to obtain a contradiction with local realism, resulting in ‘loopholes’^{13–16}. Here we report a Bell experiment that is free of any such additional assumption and thus directly tests the principles underlying Bell’s inequality. We use an event-ready scheme^{17–19} that enables the generation of robust entanglement between distant electron spins (estimated state fidelity of 0.92 ± 0.03). Efficient

sufficiently separated such that locality prevents communication between the boxes during a trial, then the following inequality holds under local realism:

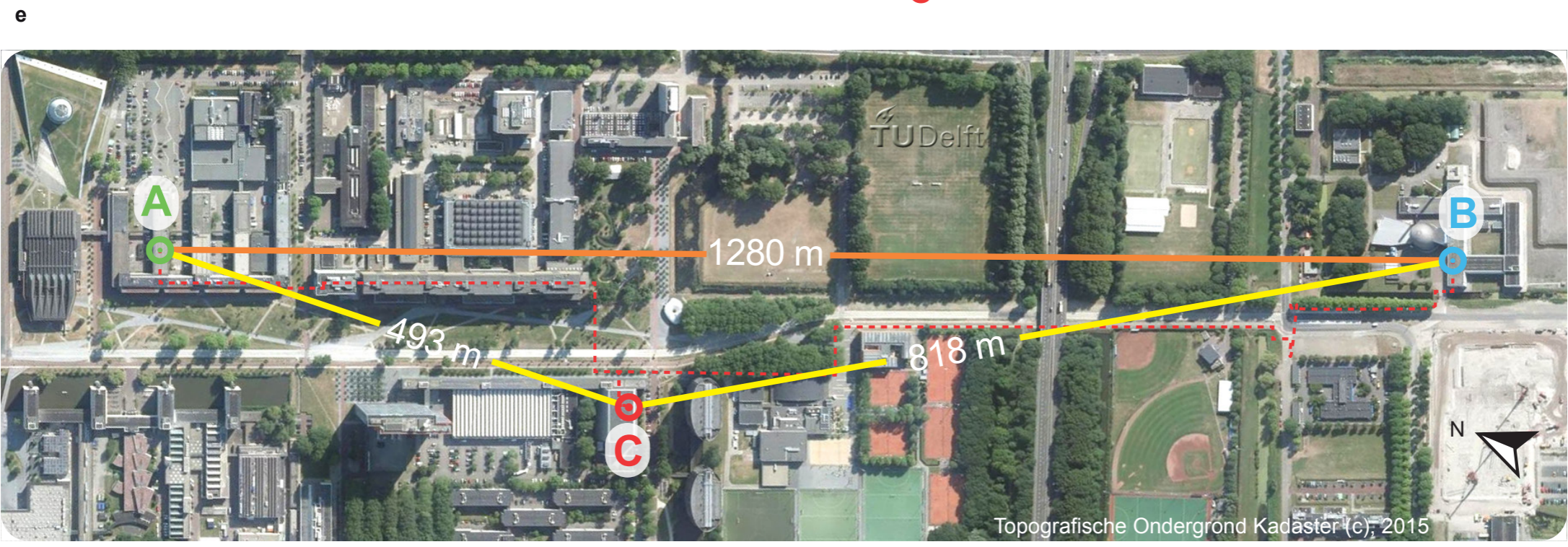
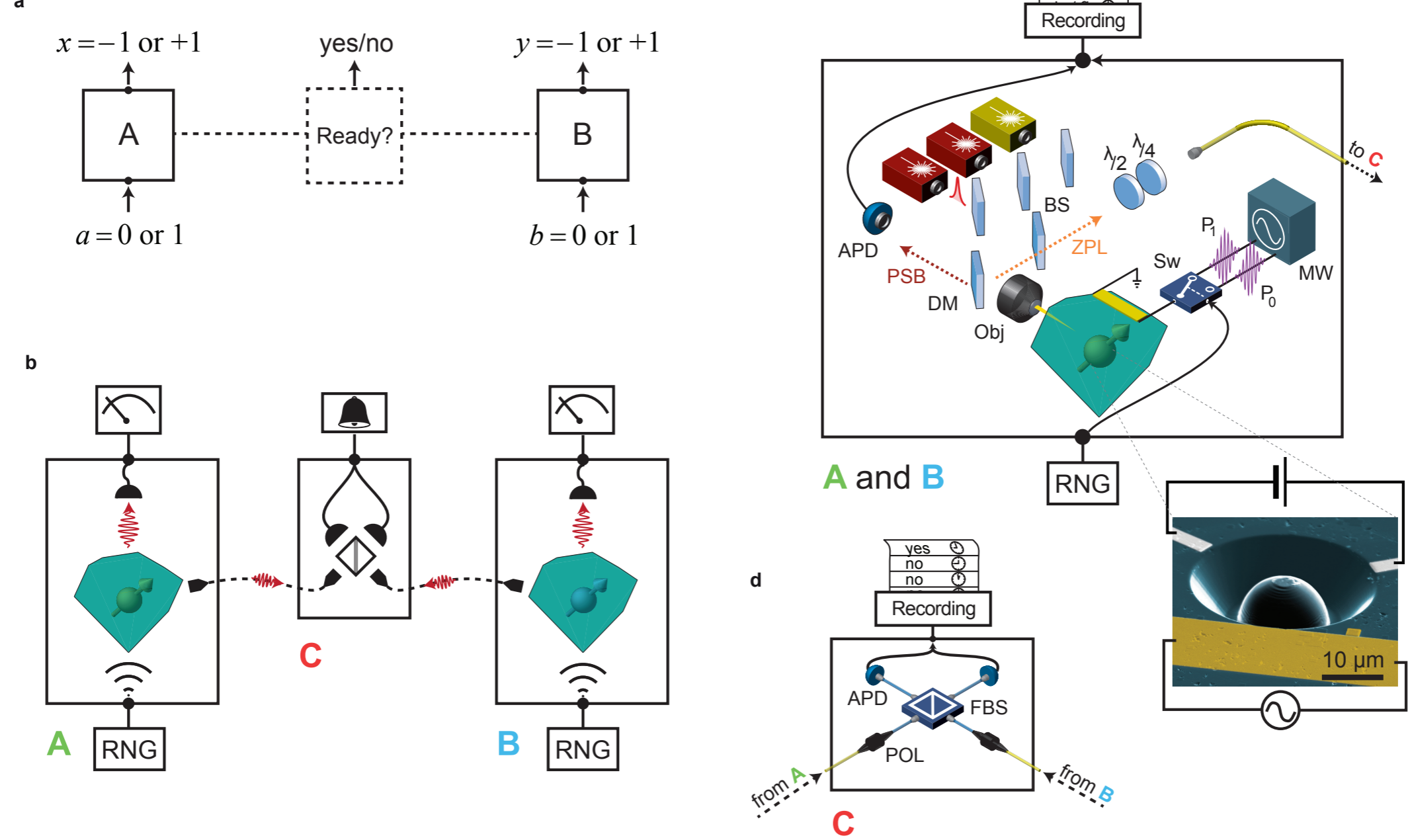
$$S = \left| \langle x \cdot y \rangle_{(0,0)} + \langle x \cdot y \rangle_{(0,1)} + \langle x \cdot y \rangle_{(1,0)} - \langle x \cdot y \rangle_{(1,1)} \right| \leq 2 \quad (1)$$

where $\langle x \cdot y \rangle_{(a,b)}$ denotes the expectation value of the product of x and y for input bits a and b . (A mathematical formulation of the concepts underlying Bell’s inequality is found in, for example, ref. 25.)

Quantum theory predicts that the Bell inequality can be significantly violated in the following setting. We add one particle, for example an electron, to each box. The spin degree of freedom of the electron forms a two-level system with eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$. For each trial, the two spins are prepared into the entangled state $|\psi^-\rangle = (|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle) / \sqrt{2}$.

$$S = \left| \langle x \cdot y \rangle_{(0,0)} + \langle x \cdot y \rangle_{(0,1)} + \langle x \cdot y \rangle_{(1,0)} - \langle x \cdot y \rangle_{(1,1)} \right| \leq 2 \quad (1)$$

Bell's inequality



QUANTUM PHYSICS

Death by experiment for local realism

A fundamental scientific assumption called local realism conflicts with certain predictions of quantum mechanics. Those predictions have now been verified, with none of the loopholes that have compromised earlier tests. SEE LETTER P.682

HOWARD WISEMAN

The world is made up of real stuff, existing in space and changing only through local interactions — this local-realism hypothesis is about the most intuitive scientific postulate imaginable. But quantum mechanics implies that it is false, as has been known for more than 50 years¹. However, brilliantly successful though quantum mechanics has been, it is still only a theory, and no definitive experiment has disproved the local-realism hypothesis — until now. On page 682 of this issue, Hensen *et al.*² report the first violation of a constraint called a Bell inequality, under conditions that prevent alternative

explanations of the experimental data. Their findings therefore rigorously reject local realism, for the first time.

Bell inequalities are named after John Bell, the physicist who discovered in 1964 that the predictions of quantum mechanics are incompatible with the local-realism hypothesis¹. There are many different ways to make this hypothesis precise³, but Hensen and colleagues' exposition basically follows Bell's original formulation, which states it as the conjunction of two other hypotheses: realism (which Bell called predetermination), essentially meaning that measurements reveal pre-existing physical properties of the world; and locality, roughly meaning that any change



50 Years Ago

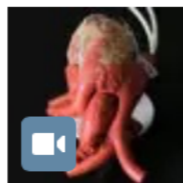
It may not be generally realized that work is in progress on the colossal project of constructing a 40-in. diameter, 300 miles long, Trans-Alpine oil pipeline to convey oil from the Adriatic to the heart of Germany ... Among the many practical problems concerned with such a project, apart from tunnelling and mechanical excavation in the high Alps, are the necessity to dredge the harbour at Trieste so that it can eventually accommodate oil tankers of 160,000 dead weight tons; setting storage tanks there on piles because available land is a rocky hill site; construction of several thousand feet of piers in the Adriatic ... Involved also in the scheme is the building of five separate pumping stations, each equipped with two 4,000-horse-power electric centrifugal pumps required to lift hundreds of thousands of tons of oil from sea-level to one of the highest points of Felber Tauern.

From *Nature* 30 October 1965





Stonehenge Begins to Yield Its Secrets



Artificial Patients, Real Learning

PAID POST: NETJETS

It's Possible: Around the World at a Moment's Notice

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SCIENCE

SCIENCE

Sorry, Einstein. Quantum Study Suggests 'Spooky Action' Is Real.

By JOHN MARKOFF OCT. 21, 2015

710 COMMENTS

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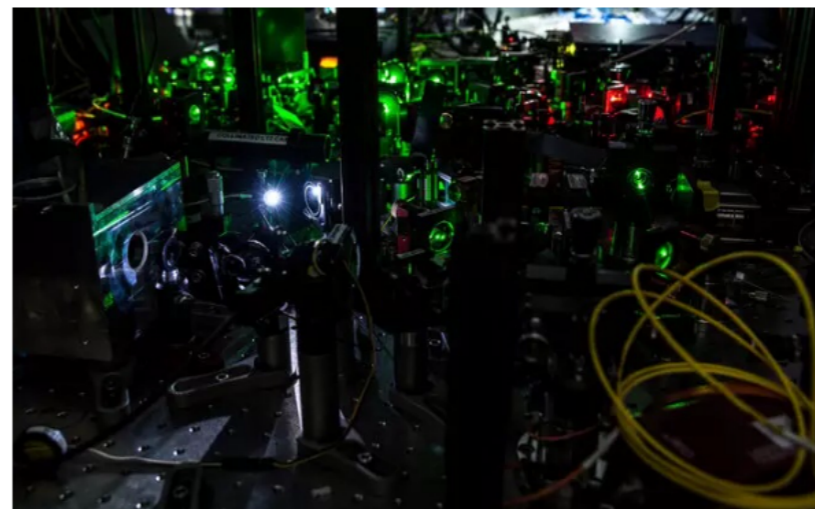
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In a landmark study, scientists at Delft University of Technology in the [Netherlands](#) reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other's behavior.

The finding is another blow to one of the bedrock

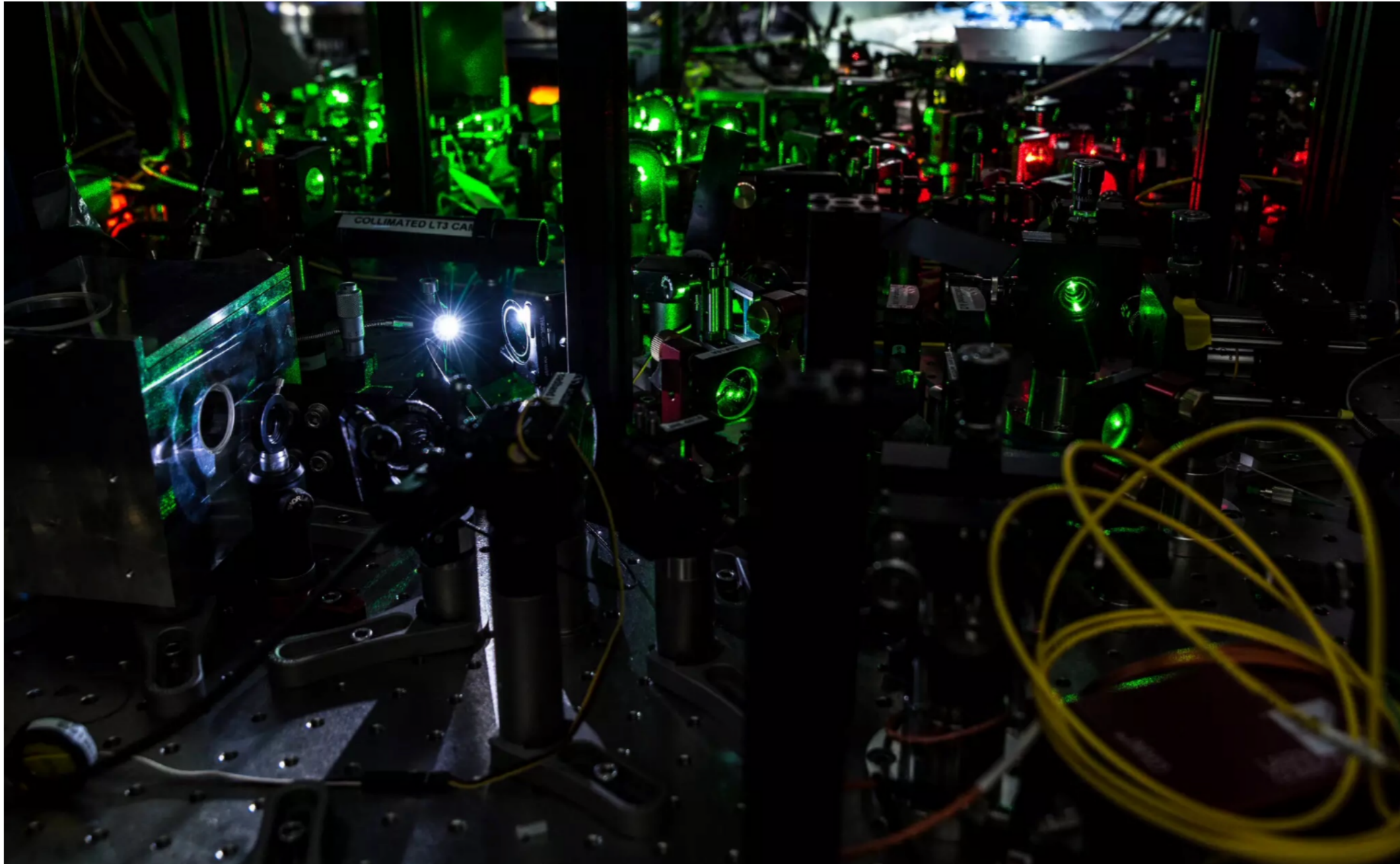


Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Frank Auperle/Delft University of Technology

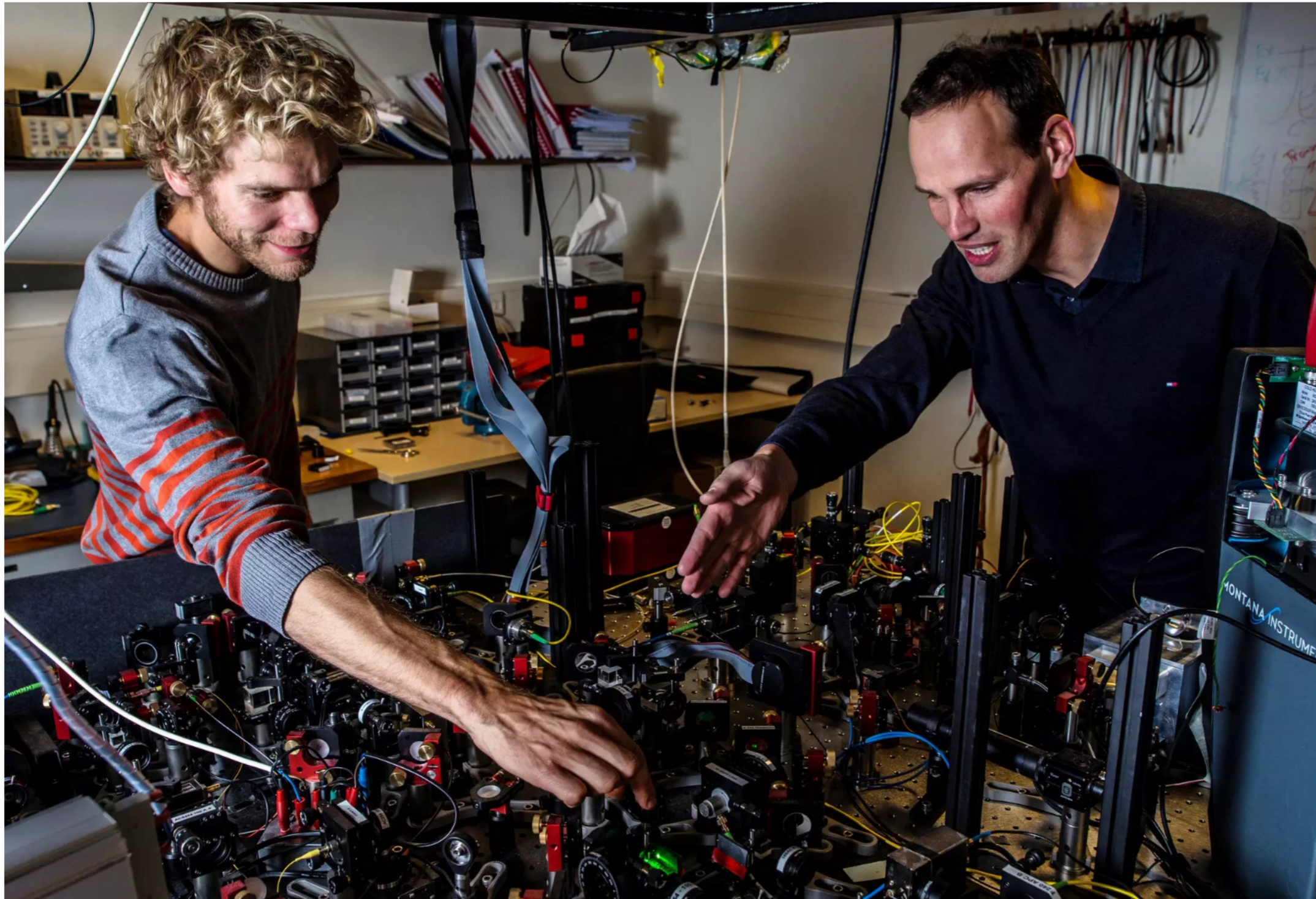
BROOKLYN
NOW PLAYING
GET TICKETS

CHRISTIE'S



Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Frank Auperle/Delft University of Technology



Bas Hensen, left, and Ronald Hanson helped show that objects apart can instantly affect each other.

Frank Auperle/Delft University of Technology

B. Hensen^{1,2}, H. Bernien^{1,2†}, A. E. Dréau^{1,2}, A. Reiserer^{1,2}, N. Kalb^{1,2}, M. S. Blok^{1,2}, J. Ruitenbergh^{1,2}, R. F. L. Vermeulen^{1,2}, R. N. Schouten^{1,2}, C. Abellán³, W. Amaya³, V. Pruneri^{3,4}, M. W. Mitchell^{3,4}, M. Markham⁵, D. J. Twitchen⁵, D. Elkouss¹, S. Wehner¹, T. H. Taminiau^{1,2} & R. Hanson^{1,2}



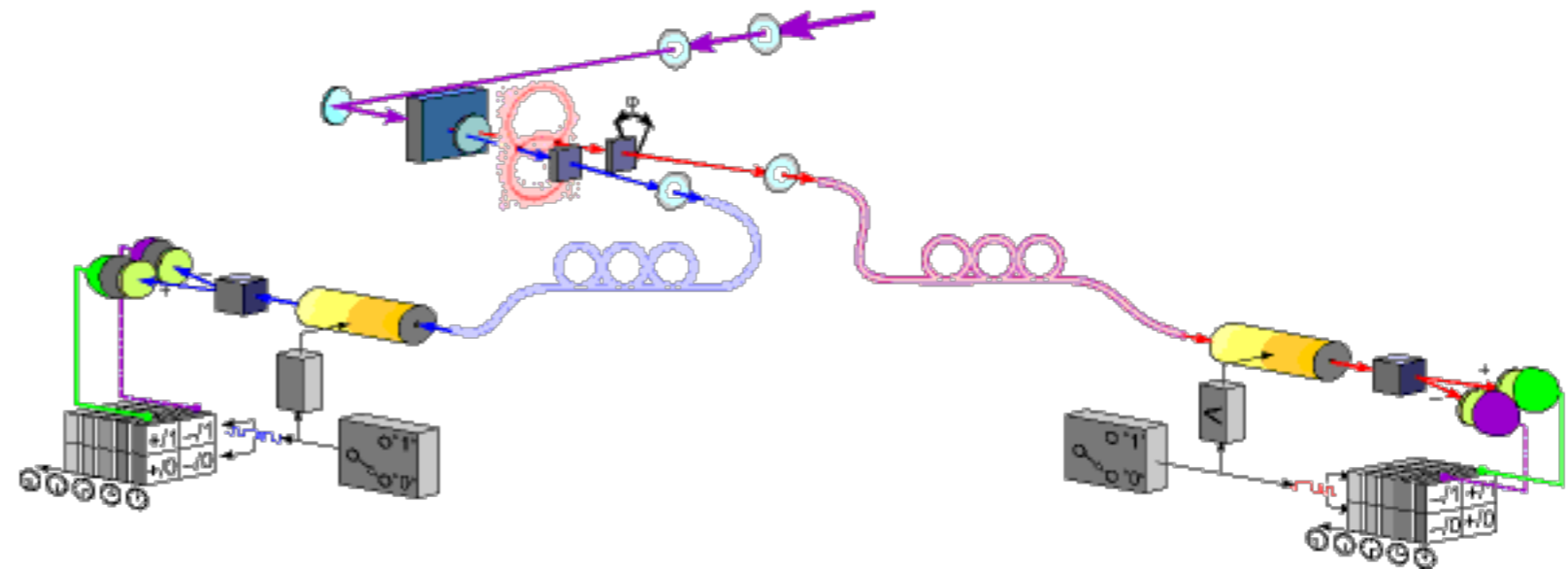
Delft co-author, mathematician Stéphanie Wehner

First loophole-free experimental violation of Bell's inequality

- John S. Bell (1964) showed that according to quantum theory, quantum systems could exhibit correlations impossible under classical physics without faster-than-light communication
- Such quantum correlations have since been observed in many laboratory experiments, but till now, always in a setting where there is a classical explanation without FTL
- In other words, they could not do the right experiment, and had to make do with surrogates (e.g.: Aspect et al. 1982; Weihs et al. 1998: ...)



Example: Weihs et al. (1998)



Les chaussettes
de M. Bertlmann
et la nature
de la réalité

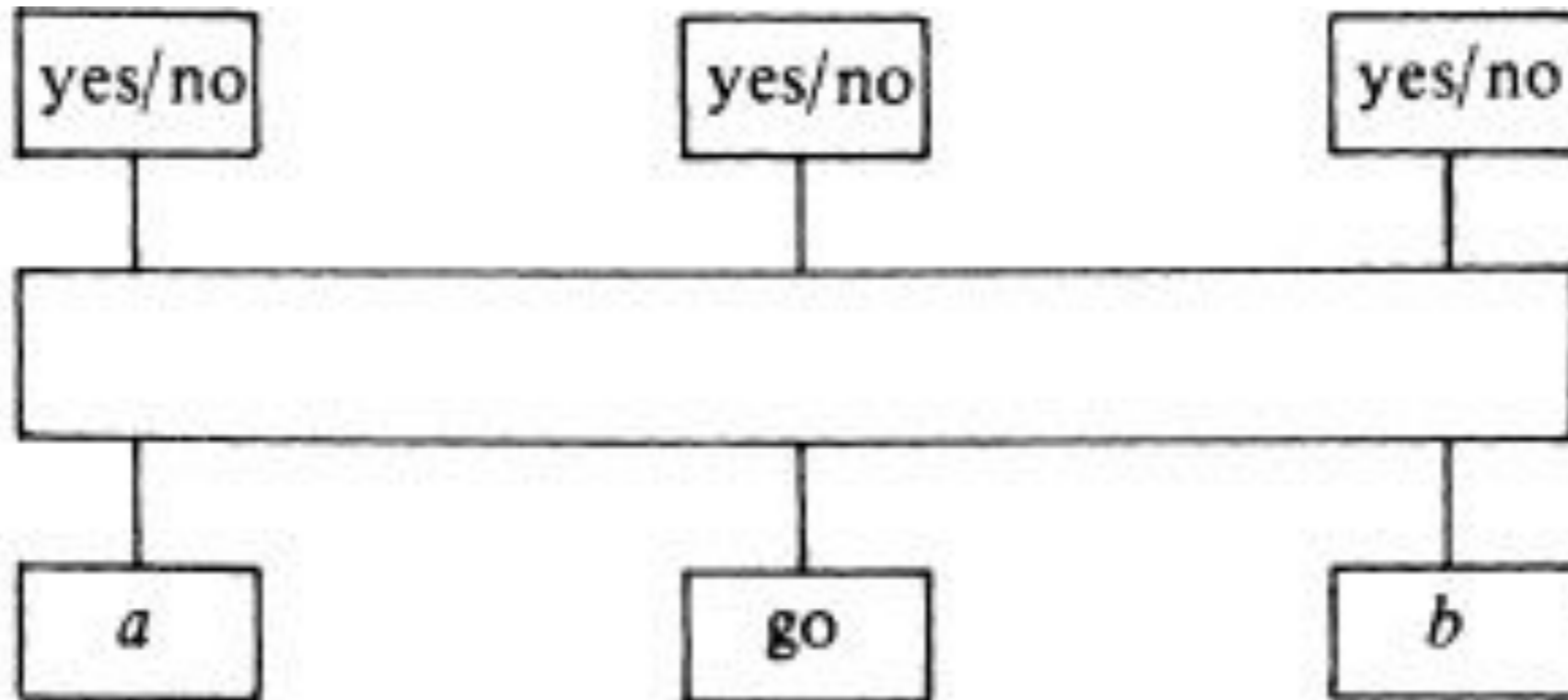
Fondation Huyot
juin 17 1980

pink → ← not pink

Bell (1981) "Bertlmann's socks and the nature of reality"

Alice Outcome

Bob Outcome



Alice Setting

Bob Setting

The Bell game

- Alice and Bob make preparations
- They are separated, and may no longer communicate
- Each is told a **setting**: “1” or “2”
- They must both now deliver an **outcome**: “*red*” or “*green*”
- Their aim: their outcomes are **equal** unless both settings are “1”, when outcomes are **different**
- Aim: outcomes r,g or g,r with settings 11; outcomes r,r or g,g with settings 12, 21, or 22

The Bell Game (continued)

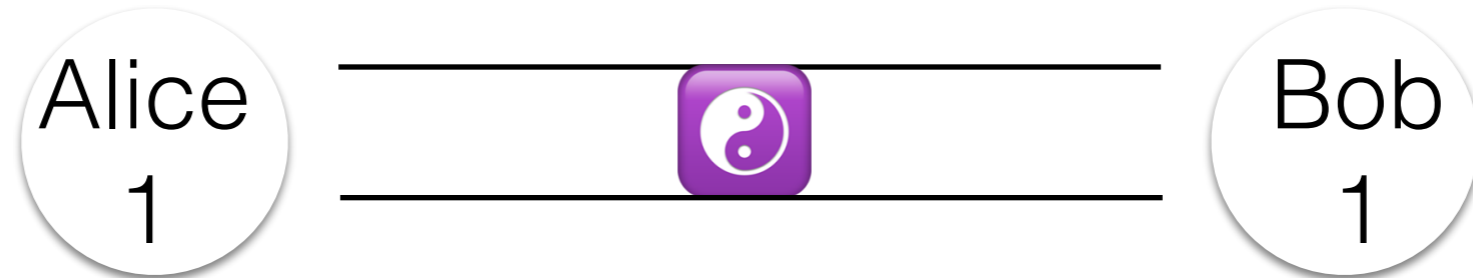
- This is repeated N times (“trials”)
- Between each trial, Alice and Bob may confer
- Their opponent can be expected to generate the settings with independent, fair, coin tosses

Optimal play (per trial)

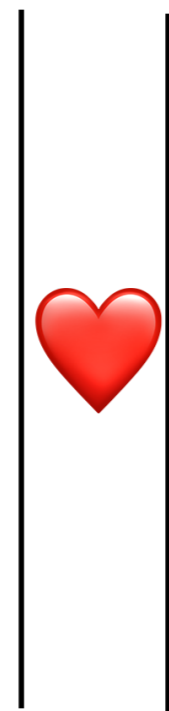
- If Alice and Bob want to use any randomisation, they might as well perform all randomisations which they either might need, in advance, while they are still together
- Given all results of any randomisations, their strategy specifies an “instruction set”: colours for Alice for settings 1 and 2, colours for Bob for settings 1 and 2
- There are exactly 16 different instruction sets
- Let’s take a look at some of them ...

Question: can you colour the four balls **green** and **red**, so that

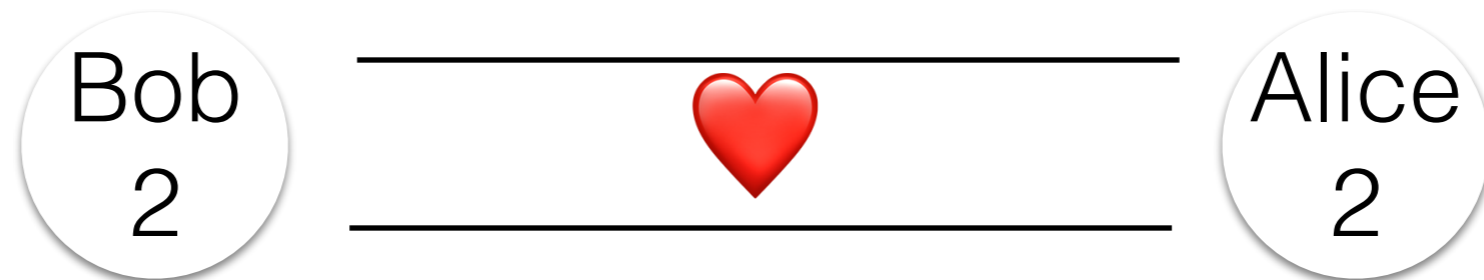
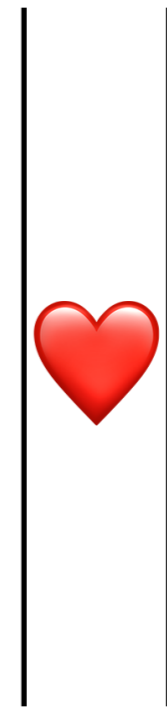
the two on top have different colours ☯



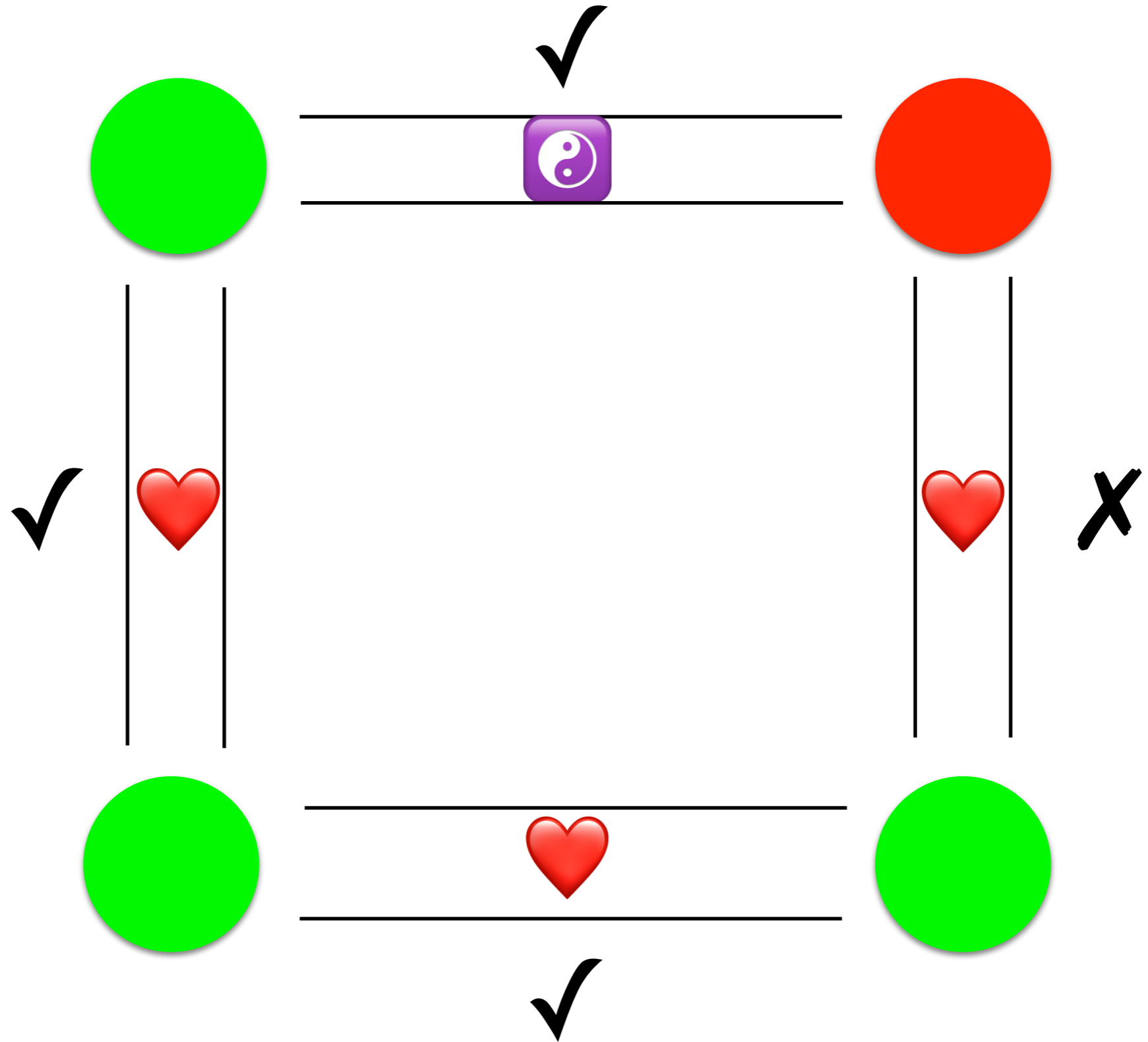
the two on the left have the same colour ❤

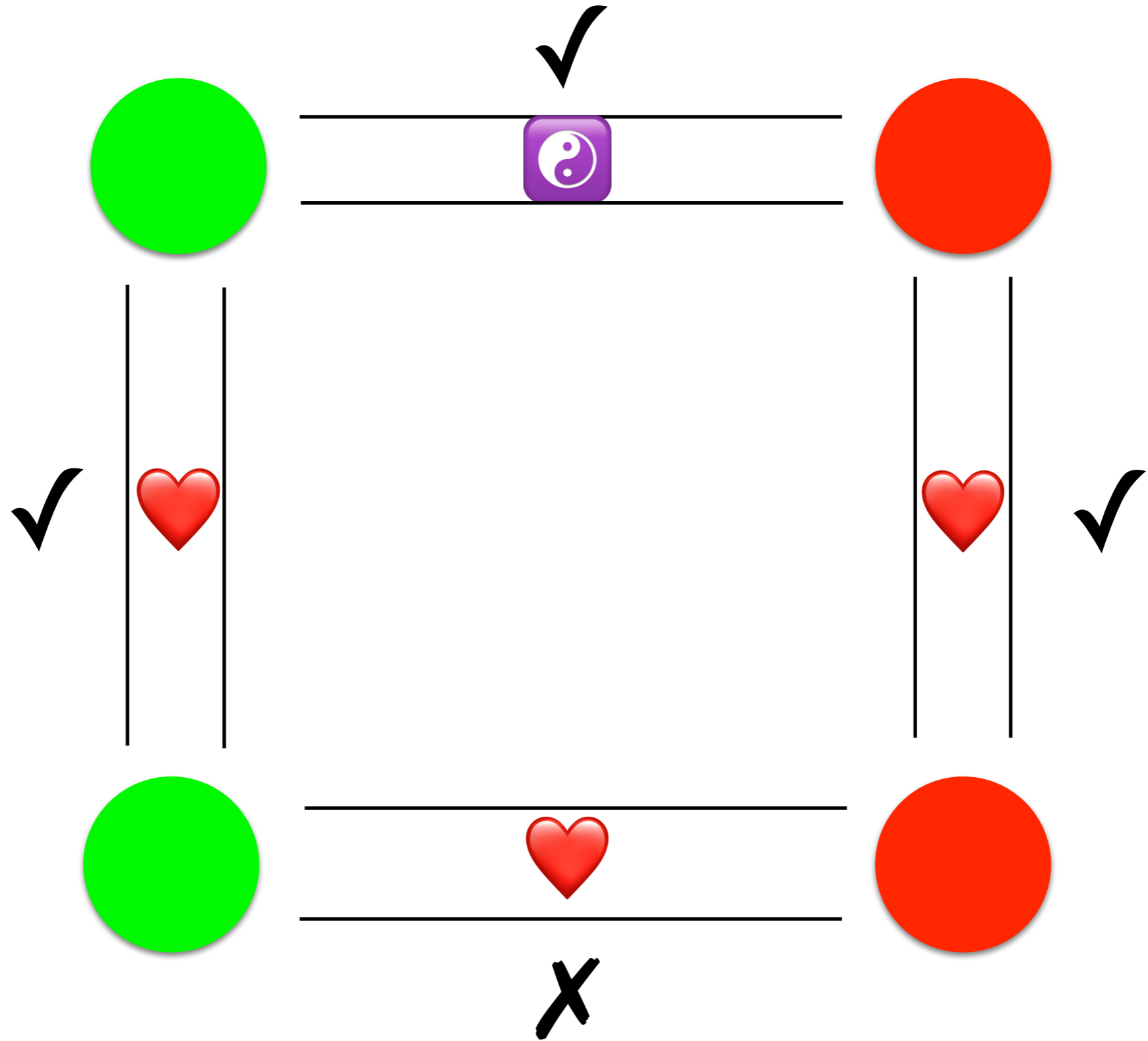


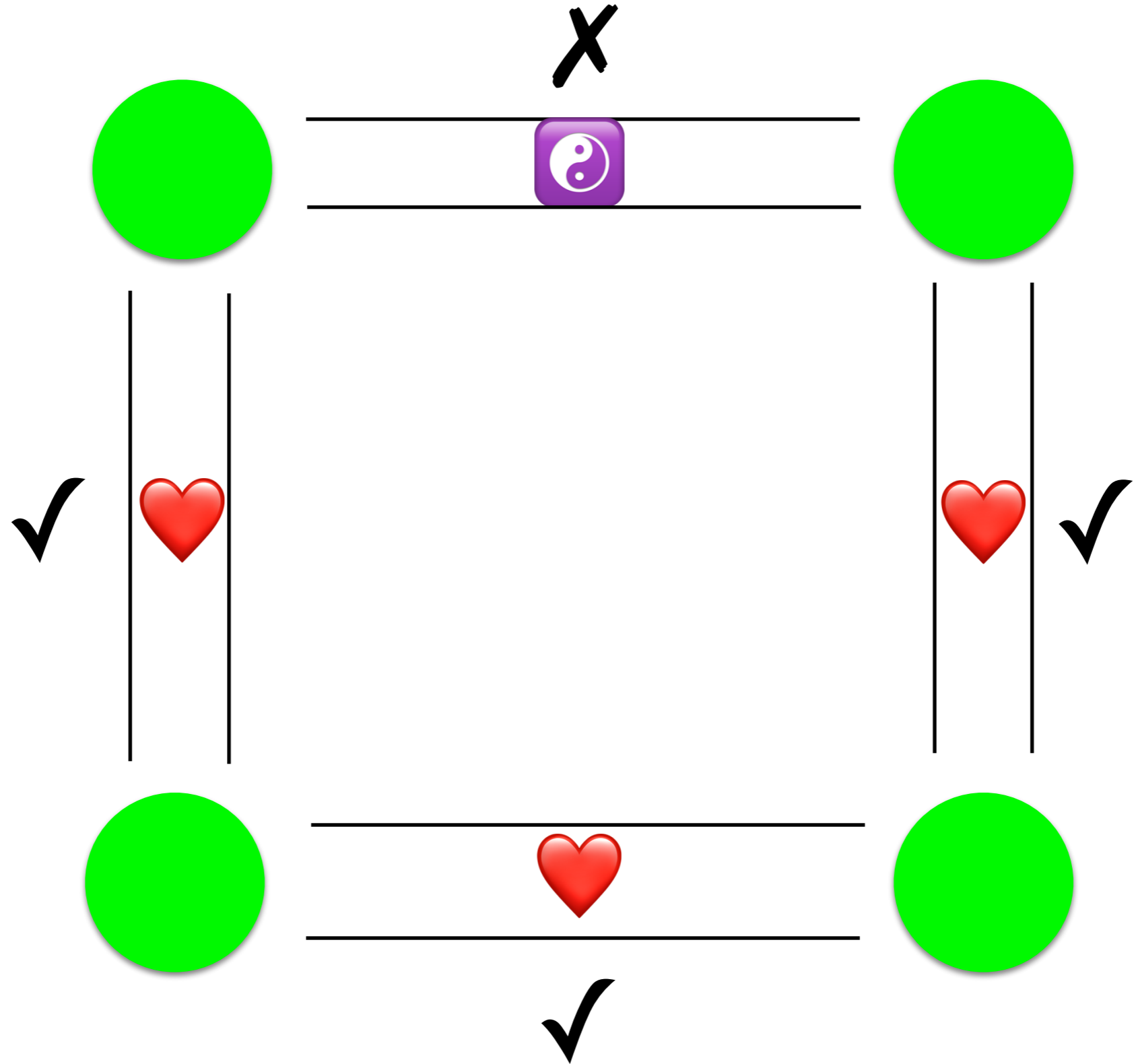
the two on the right have the same colour ❤

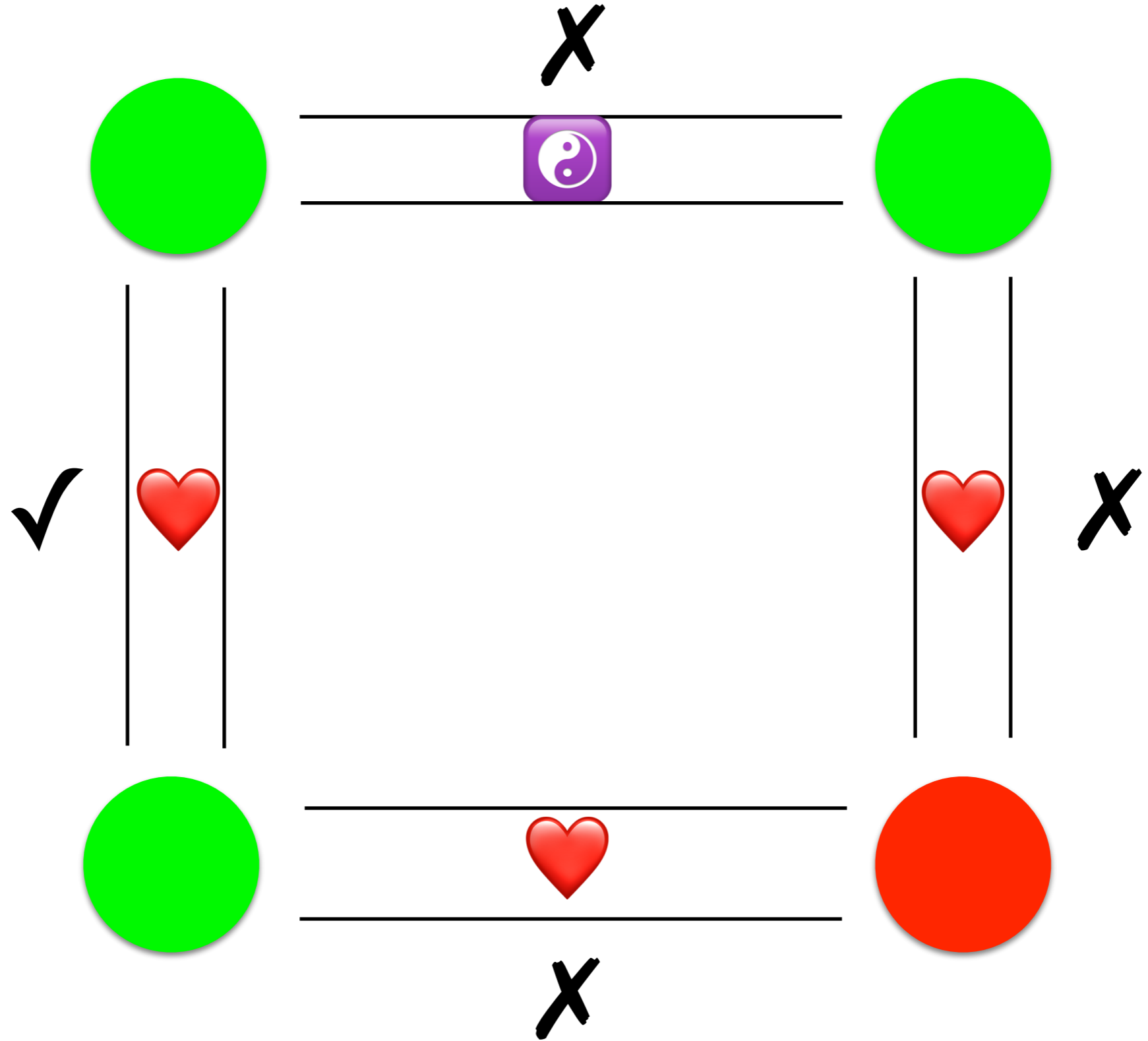


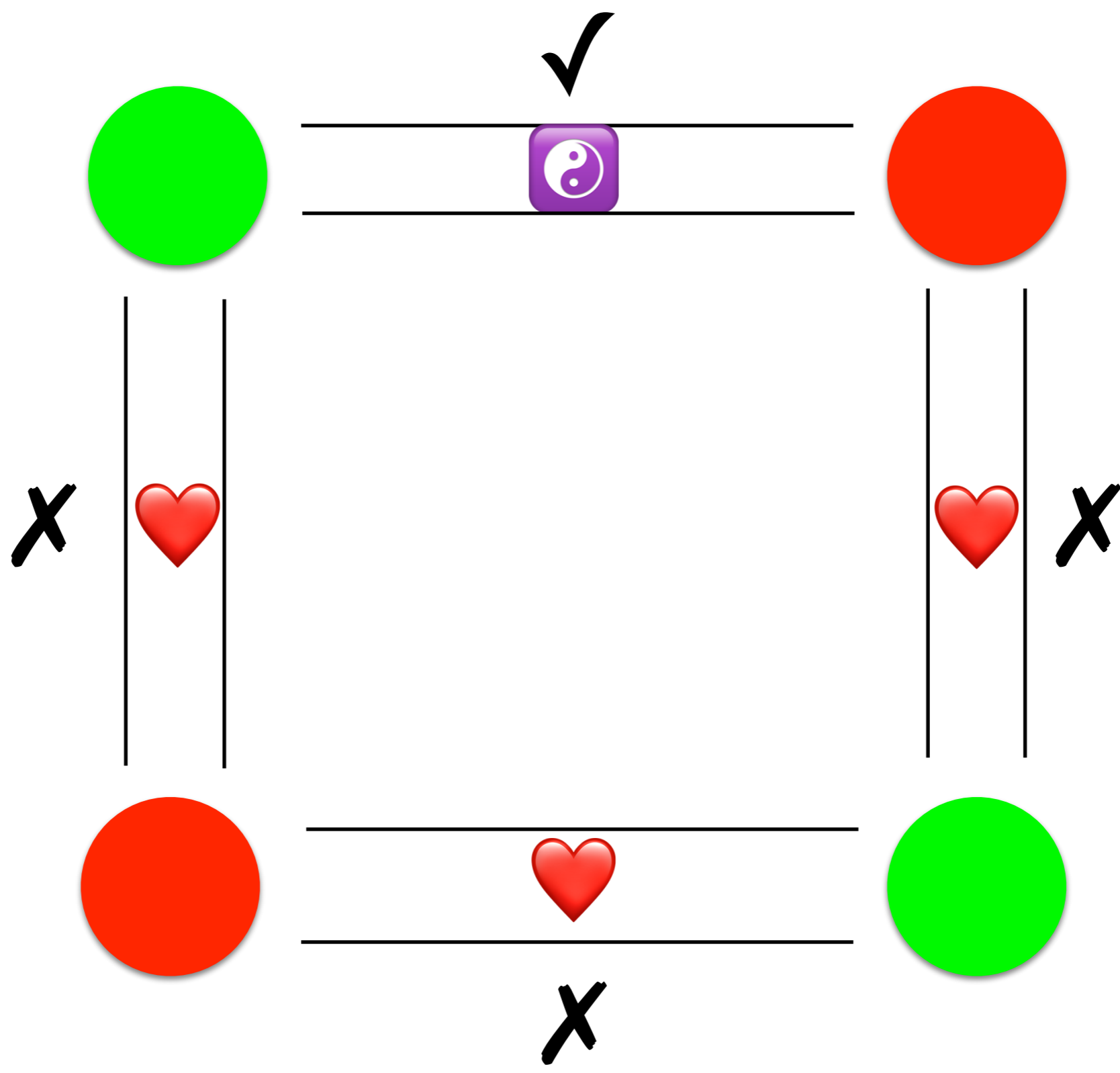
the two on the bottom have the same colour ❤

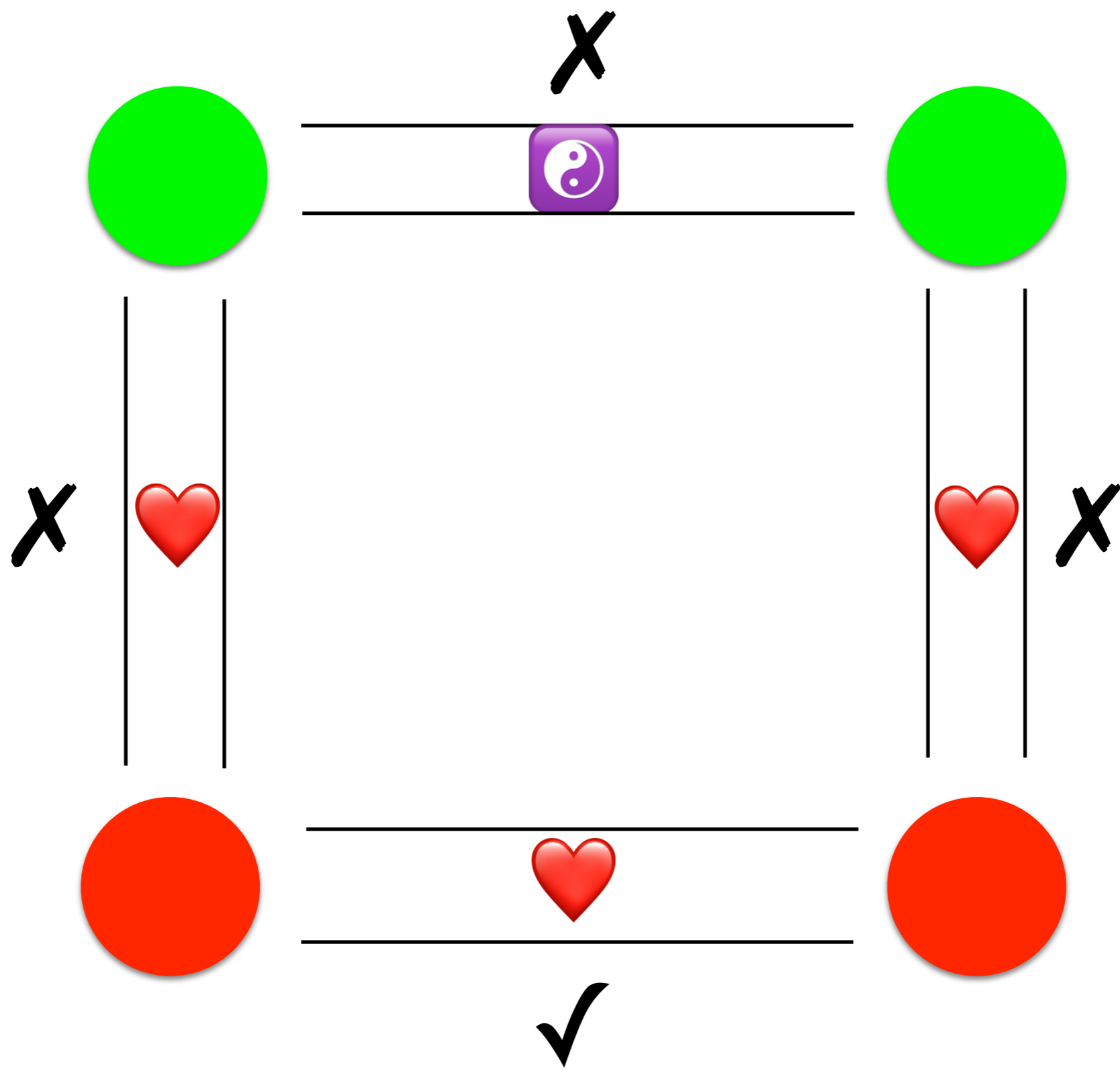






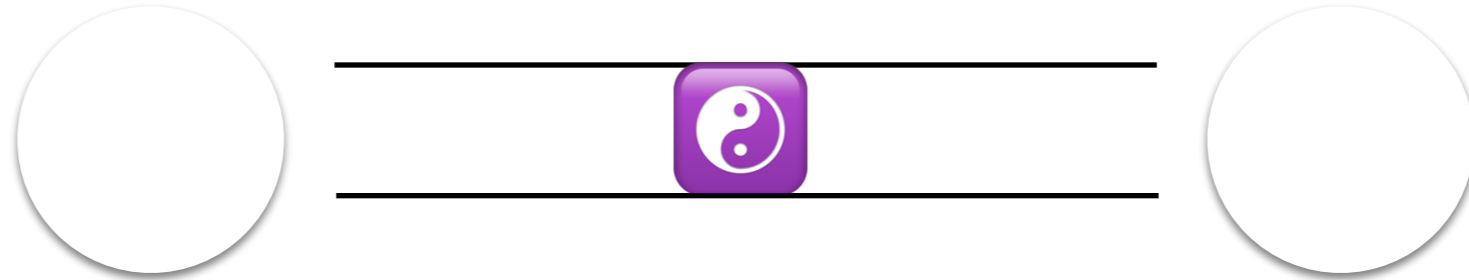




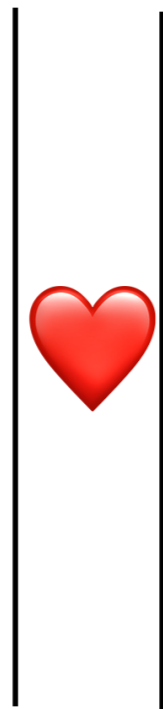


Question: can you colour the four balls **green** and **red**, so that

the two on top have different colours ☯

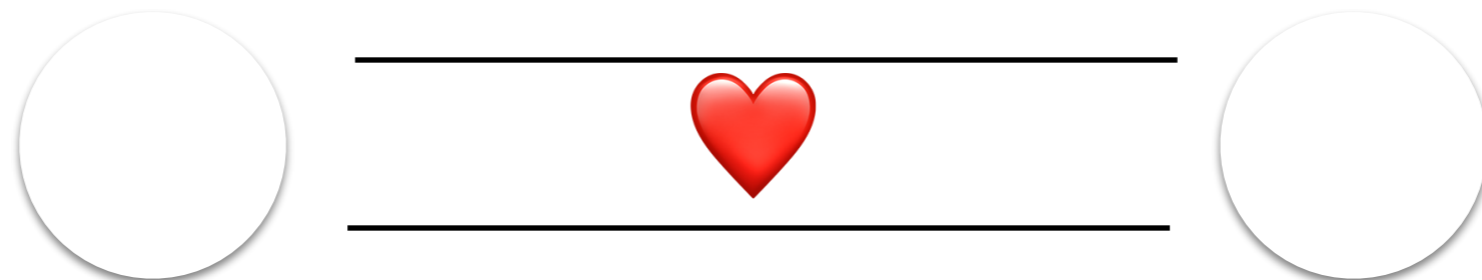
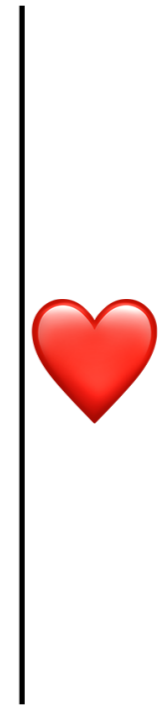


the two on the
left have the
same colour ❤️



Answer:
No you can't.
Either one
or three
failures

the two on the
right have the
same colour ❤️



the two on the bottom have the same colour ❤️

Optimal play for Alice and Bob

- 8 = 2 x 4 instruction sets deliver 3 successes, 1 failure, as we run through the four setting pairs (11 = top, 12 = left, 21 = right, 22 = bottom)
- The other 8 deliver 3 failures, 1 success
- Choosing 1 of the first 8 **uniformly at random** seems smart
- Could it be that when playing many rounds of the game, it is better to vary the strategy, possibly depending on results obtained so far??

Bell game results in Delft

- $N = 245$
- Success rate: 80%
- Optimal rate under “local realism” 75%
- Optimal rate under “quantum mechanics” 85%

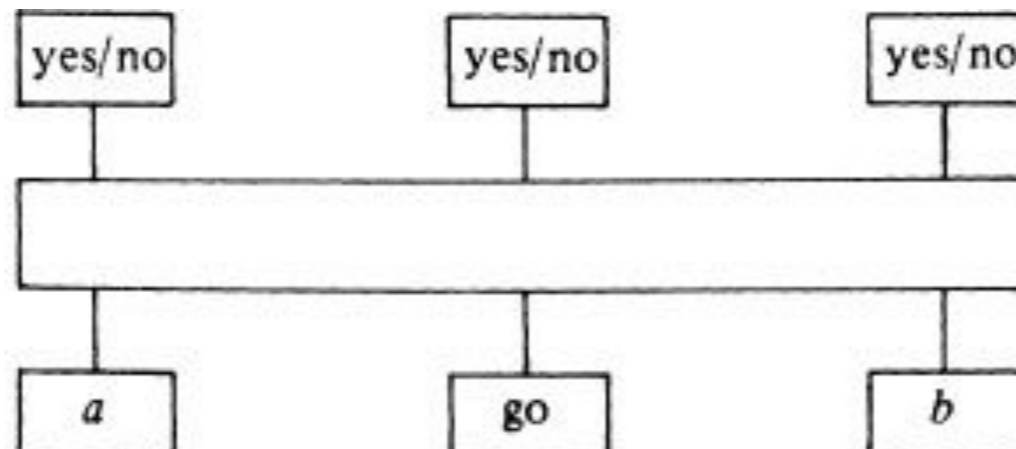
(why can't QM do better?)

Delft Bell results in round numbers

- 75% of 240 is 180
- 80% of 240 is 192
- Binomial variance $N = 240$, $p = 3/4$ is $240 \times 3/4 \times 1/4 = 45$ not far from $49 = 7 \times 7$
- $192 - 180 = 12 = \text{approx } 2 \text{ standard deviations}$
- Actual result: $N = 245$, # successes = 196
- $\text{Prob}(\text{Binomial}(245, 3/4) \geq 196) = 0.039$

There is no gain in strategies which use memory and time

- First such results obtained by Gill (2001) using martingale theory; rewrite usual “combination of four correlations” as final result of a game
- My aim: design a bet against someone who claims to be able to simulate the quantum correlations with (classically) networked classical computers

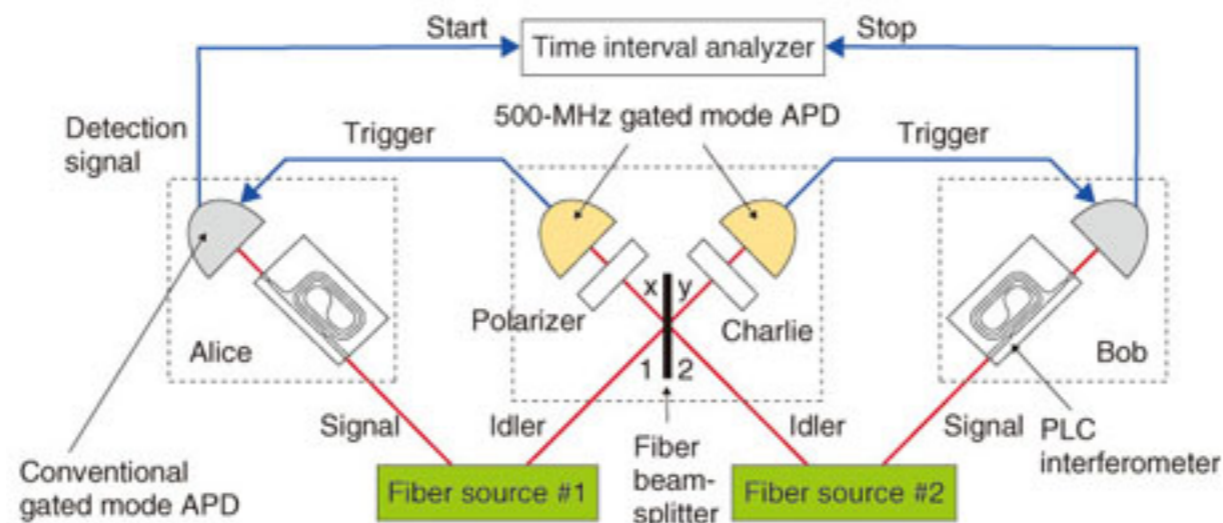


Martingale result

- The probability of at least 196 successes in 245 trials is at most $\text{Prob}(\text{Binomial}(245, 3/4) \geq 196) = 0.039$, whatever strategy is used (possibly time dependent, possibly dynamic)
- What is essential: settings are chosen repeatedly completely at random

Delft innovation: use entanglement swapping

- Photons leave each spin system and (hopefully) reach central location and interact there
- Sometimes they are both detected after interaction



Algebra (abracadabra?)

$$(00 + 11)(00 + 11) = 0000 + 0011 + 1100 + 1111$$

$$= 0(00)0 + 0(01)1 + 1(10)0 + 1(11)1$$

$$= 11 + 22 + 33 + 44$$

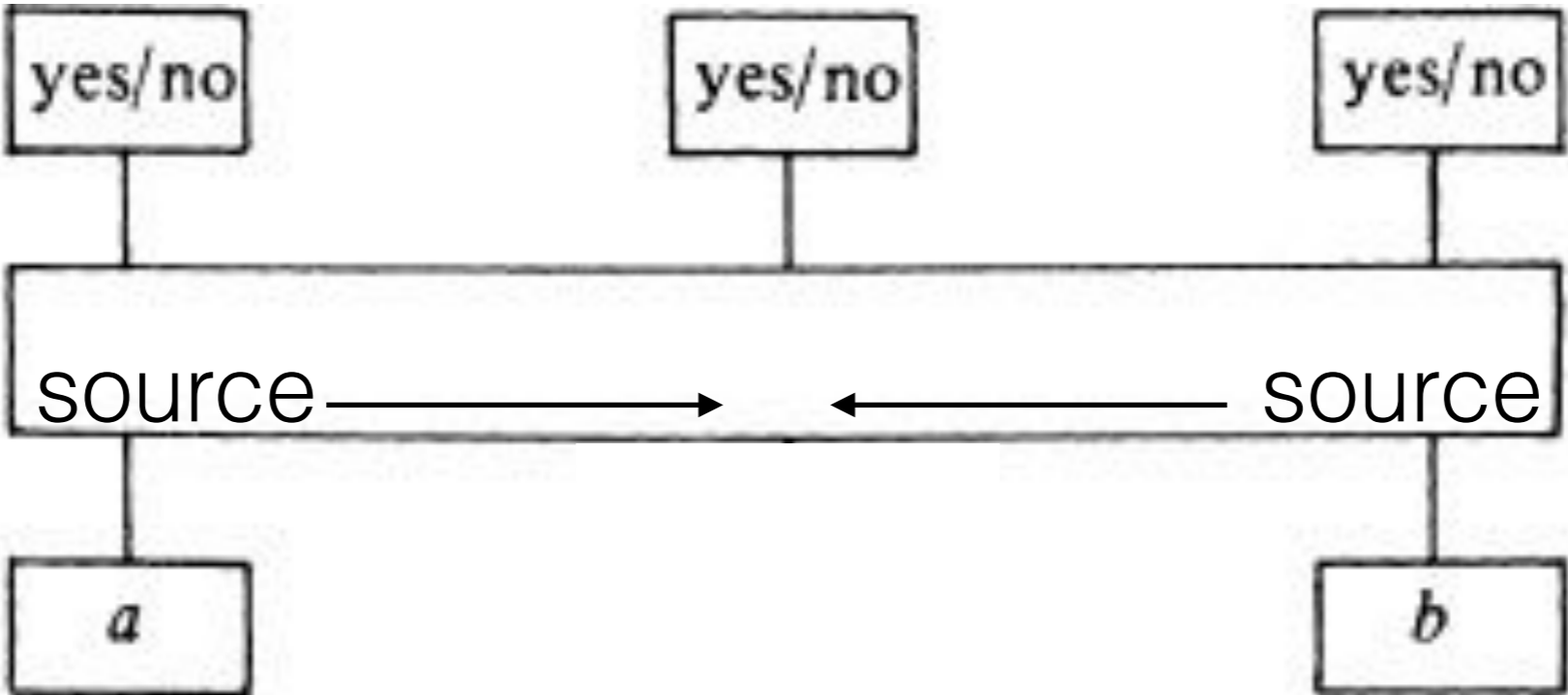
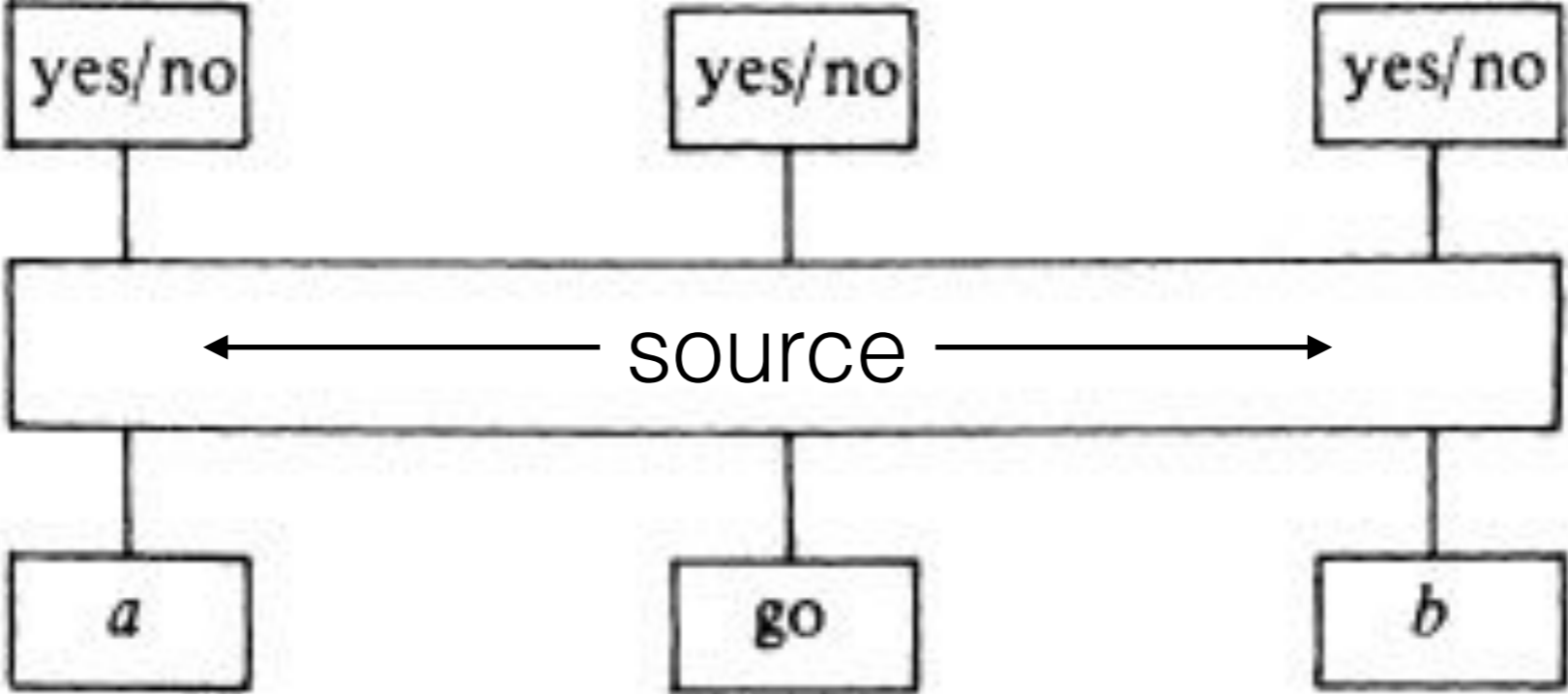
$$11 + 44 = 1((1 + 4) + (1 - 4)) + 4((1 + 4) - (1 - 4))$$

$$=(1 + 4)(1 + 4) + (1 - 4)(1 - 4)$$

$$(00 + 11)(00 + 11)$$

$$= (00 + 11)(00 + 11) + (00 - 11)(00 - 11) + (01 + 10)(01 + 10) + (01 - 10)(01 - 10)$$

Traditional



New

Another experiment

PRL 115, 250402 (2015)

 Selected for a **Viewpoint** in *Physics*
PHYSICAL REVIEW LETTERS

week ending
18 DECEMBER 2015



Strong Loophole-Free Test of Local Realism*

Lynden K. Shalm,^{1,†} Evan Meyer-Scott,² Bradley G. Christensen,³ Peter Bierhorst,¹ Michael A. Wayne,^{3,4} Martin J. Stevens,¹ Thomas Gerrits,¹ Scott Glancy,¹ Deny R. Hamel,⁵ Michael S. Allman,¹ Kevin J. Coakley,¹ Shellee D. Dyer,¹ Carson Hodge,¹ Adriana E. Lita,¹ Varun B. Verma,¹ Camilla Lambrocco,¹ Edward Tortorici,¹ Alan L. Migdall,^{4,6} Yanbao Zhang,² Daniel R. Kumor,³ William H. Farr,⁷ Francesco Marsili,⁷ Matthew D. Shaw,⁷ Jeffrey A. Stern,⁷ Carlos Abellán,⁸ Waldimar Amaya,⁸ Valerio Pruneri,^{8,9} Thomas Jennewein,^{2,10} Morgan W. Mitchell,^{8,9} Paul G. Kwiat,³ Joshua C. Bienfang,^{4,6} Richard P. Mirin,¹ Emanuel Knill,¹ and Sae Woo Nam^{1,‡}

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We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using fast random number generators and high-speed polarization measurements. A high-quality polarization-entangled source of photons, combined with high-efficiency, low-noise, single-photon detectors, allows us to make measurements without requiring any fair-sampling assumptions. Using a hypothesis test, we compute p values as small as 5.9×10^{-9} for our Bell violation while maintaining the spacelike separation of our events. We estimate the degree to which a local realistic system could predict our measurement choices. Accounting for this predictability, our smallest adjusted p value is 2.3×10^{-7} . We therefore reject the hypothesis that local realism governs our experiment.

Yet another ...

PRL **115**, 250401 (2015)

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PHYSICAL REVIEW LETTERS

week ending
18 DECEMBER 2015



Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons

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(Received 10 November 2015; published 16 December 2015)

Local realism is the worldview in which physical properties of objects exist independently of measurement and where physical influences cannot travel faster than the speed of light. Bell's theorem states that this worldview is incompatible with the predictions of quantum mechanics, as is expressed in Bell's inequalities. Previous experiments convincingly supported the quantum predictions. Yet, every experiment requires assumptions that provide loopholes for a local realist explanation. Here, we report a Bell test that closes the most significant of these loopholes simultaneously. Using a well-optimized source of entangled photons, rapid setting generation, and highly efficient superconducting detectors, we observe a violation of a Bell inequality with high statistical significance. The purely statistical probability of our results to occur under local realism does not exceed 3.74×10^{-31} , corresponding to an 11.5 standard

Tiny violation, huge significance

- Giustina et al. (Vienna): success rate 75.00073%, $N = 3503$ million
- Shalm et al. (NIST, Boulder, Co): success rate 75.00142%, $N = 177$ million
- p-values ...
- These are both “traditional” types of the experiment

Conclusion

- We need better experiments still ...
- They will certainly need statistics

Want to know more?

- <http://www.slideshare.net/gill1109/epidemiology-meets-quantum-statistics-causality-and-bells-theorem>
- <http://www.math.leidenuniv.nl/~gill>
- Survey paper in *Statistical Science*



Statistical Science
2014, Vol. 29, No. 4, 512–528
DOI: 10.1214/14-STS490
© Institute of Mathematical Statistics, 2014

Statistics, Causality and Bell's Theorem

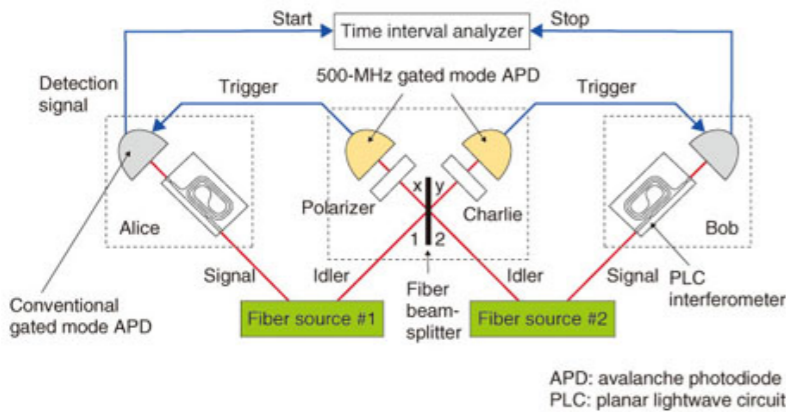
Richard D. Gill

Abstract. Bell's [*Physics* 1 (1964) 195–200] theorem is popularly supposed to establish the nonlocality of quantum physics. Violation of Bell's inequality in experiments such as that of Aspect, Dalibard and Roger [*Phys. Rev. Lett.* 49 (1982) 1804–1807] provides empirical proof of nonlocality in the real world. This paper reviews recent work on Bell's theorem, linking it to issues in causality as understood by statisticians. The paper starts with a proof of a strong, finite sample, version of Bell's inequality and thereby also of Bell's theorem, which states that quantum theory is incompatible with the conjunction of three formerly uncontroversial physical principles, here referred to as *locality*, *realism* and *freedom*.





Postscript



I cannot say that action at a distance is required in physics. But I cannot say that you can get away with no action at a distance. You cannot separate off what happens in one place with what happens at another – John Bell

<https://www.youtube.com/watch?v=V8CCfOD1iu8>

Nature produces chance events (irreducibly chance-like!) which can occur at widely removed spatial locations without anything propagating from point to point along any path joining those locations. ... The chance-like character of these effects prevents any possibility of using this form of non locality to communicate, thereby saving from contradiction one of the fundamental principles of relativity theory according to which no communication can travel faster than the speed of light – Nicolas Gisin

Quantum Chance: Nonlocality, Teleportation and Other Quantum Marvels. Springer, 2014