```
R version 3.6.0 (2019-04-26) -- "Planting of a Tree"
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Platform: x86_64-apple-darwin15.6.0 (64-bit)

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[R.app GUI 1.70 (7657) x86_64-apple-darwin15.6.0]

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>
> ############ VIENNA ############
>
> table11 <- matrix(c(141439,    73391,    76224,    875392736),
+     2, 2, byrow = TRUE,
+     dimnames = list(Alice = c("d", "n"), Bob = c("d", "n")))
> table12 <- matrix(c(146831,    67941,    326768,    874976534),
+     2, 2, byrow = TRUE,
+     dimnames = list(Alice = c("d", "n"), Bob = c("d", "n")))
> table21 <- matrix(c(158338,    425067,    58742,    875239860),
+     2, 2, byrow = TRUE,
+     dimnames = list(Alice = c("d", "n"), Bob = c("d", "n")))
> table22 <- matrix(c(8392,    576445,    463985,    874651457),
+     2, 2, byrow = TRUE,
+     dimnames = list(Alice = c("d", "n"), Bob = c("d", "n")))
>
> table11
      Bob
Alice      d         n
    d 141439     73391
    n  76224 875392736
> table12
      Bob
Alice      d         n
    d 146831     67941
    n 326768 874976534
> table21
      Bob
Alice      d         n
    d 158338     425067
    n  58742 875239860
> table22
      Bob
Alice      d         n
    d   8392     576445
    n 463985 874651457
>
> tables <- cbind(as.vector(t(table11)), as.vector(t(table12)), as.vector(t(table21)), as.vector(t(table22)))
> dimnames(tables) = list(outcomes = c("dd", "dn", "nd", "nn"),
+                   settings = c(11, 12, 21, 22))
> tables
        settings
outcomes        11        12        21        22
      dd    141439    146831    158338      8392
      dn     73391     67941    425067    576445
      nd     76224    326768     58742    463985
      nn 875392736 874976534 875239860 874651457
>
> Ns <- apply(tables, 2, sum)
> Ns
       11        12        21        22
875683790 875518074 875882007 875700279
>
> rawProbsMat <- tables / outer(rep(1,4), Ns)
> rawProbsMat
        settings
outcomes           11           12           21           22
      dd 1.615183e-04 1.677076e-04 1.807755e-04 9.583188e-06
      dn 8.380993e-05 7.760091e-05 4.853017e-04 6.582675e-04
      nd 8.704512e-05 3.732282e-04 6.706611e-05 5.298445e-04
      nn 9.996676e-01 9.993815e-01 9.992669e-01 9.988023e-01
>
> VecNames <- as.vector(t(outer(colnames(rawProbsMat), rownames(rawProbsMat), paste, sep = "")))
```

```
> VecNames
 [1] "11dd" "11dn" "11nd" "11nn" "12dd" "12dn" "12nd" "12nn" "21dd" "21dn" "21nd" "21nn" "22dd" "22dn" "22nd" "22nn"
>
> rawProbsVec <- as.vector(rawProbsMat)
> names(rawProbsVec) <- VecNames
>
> VecNames
 [1] "11dd" "11dn" "11nd" "11nn" "12dd" "12dn" "12nd" "12nn" "21dd" "21dn" "21nd" "21nn" "22dd" "22dn" "22nd" "22nn"
> rawProbsVec
        11dd         11dn         11nd         11nn         12dd         12dn         12nd         12nn         21dd
1.615183e-04 8.380993e-05 8.704512e-05 9.996676e-01 1.677076e-04 7.760091e-05 3.732282e-04 9.993815e-01 1.807755e-04
        21dn         21nd         21nn         22dd         22dn         22nd         22nn
4.853017e-04 6.706611e-05 9.992669e-01 9.583188e-06 6.582675e-04 5.298445e-04 9.988023e-01
>
> Aplus <- c(1, 1, 0, 0)
> Aminus <- - Aplus
> Bplus <- c(1, 0, 1, 0)
> Bminus <- - Bplus
> zero <- c(0, 0, 0, 0)
> NSa1 <- c(Aplus, Aminus, zero, zero)
> NSa2 <- c(zero, zero, Aplus, Aminus)
> NSb1 <- c(Bplus, zero, Bminus, zero)
> NSb2 <- c(zero, Bplus, zero, Bminus)
> NS <- cbind(NSa1 = NSa1, NSa2 = NSa2, NSb1 = NSb1, NSb2 = NSb2)
> rownames(NS) <- VecNames
> NS
     NSa1 NSa2 NSb1 NSb2
11dd    1    0    1    0
11dn    1    0    0    0
11nd    0    0    1    0
11nn    0    0    0    0
12dd   -1    0    0    1
12dn   -1    0    0    0
12nd    0    0    0    1
12nn    0    0    0    0
21dd    0    1   -1    0
21dn    0    1    0    0
21nd    0    0   -1    0
21nn    0    0    0    0
22dd    0   -1    0   -1
22dn    0   -1    0    0
22nd    0    0    0   -1
22nn    0    0    0    0
>
> # "The number of valid trials is N = 3 502 784 150"
> sum(Ns)
[1] 3502784150
>
> cov11 <- diag(rawProbsMat[ , "11"]) - outer(rawProbsMat[ , "11"], rawProbsMat[ , "11"])
> cov12 <- diag(rawProbsMat[ , "12"]) - outer(rawProbsMat[ , "12"], rawProbsMat[ , "12"])
> cov21 <- diag(rawProbsMat[ , "21"]) - outer(rawProbsMat[ , "21"], rawProbsMat[ , "21"])
> cov22 <- diag(rawProbsMat[ , "22"]) - outer(rawProbsMat[ , "22"], rawProbsMat[ , "22"])
>
> Cov <- matrix(0, 16, 16)
> rownames(Cov) <- VecNames
> colnames(Cov) <- VecNames
> Cov[1:4, 1:4] <- cov11/Ns["11"]
> Cov[5:8, 5:8] <- cov12/Ns["12"]
> Cov[9:12, 9:12] <- cov21/Ns["21"]
> Cov[13:16, 13:16] <- cov22/Ns["22"]
>
> InvCovNN <- solve(covNN)
>
> J <- c(c(1, 0, 0 ,0), c(0, -1, 0 ,0), c(0, 0, -1, 0), c(-1, 0, 0, 0))
> names(J) <- VecNames
> sum(J * rawProbsVec)
[1] 7.26814e-06
>
> varJ <- t(J) %*% Cov %*% J
> sum(J * rawProbsVec) / sqrt(varJ)
         [,1]
[1,] 12.10426
> pnorm(sum(J * rawProbsVec) / sqrt(varJ), lower.tail = FALSE, log = TRUE)/log(10)
          [,1]
[1,] -33.29985
>
> covNN <- t(NS) %*% Cov %*% NS
> covJN <- t(J) %*% Cov %*% NS
> covNJ <- t(covJN)
> varJ - covJN %*% InvCovNN %*% covNJ
             [,1]
[1,] 3.494185e-13
> varJ
             [,1]
[1,] 3.605539e-13
> sqrt(varJ / (varJ - covJN %*% InvCovNN %*% covNJ))
         [,1]
```

```
[1,] 1.015809
> covJN %*% InvCovNN
           NSa1        NSa2      NSb1        NSb2
[1,] 0.02133916 0.002669602 0.02012022 0.002698914
>
> Jopt <- J - covJN %*% InvCovNN %*% t(NS)
> Jopt
          11dd        11dn        11nd 11nn       12dd       12dn        12nd 12nn       21dd        21dn       21nd
[1,] 0.9585406 -0.02133916 -0.02012022    0 0.01864025 -0.9786608 -0.002698914    0 0.01745062 -0.002669602 -0.9798798
        21nn       22dd        22dn        22nd 22nn
[1,]    0 -0.9946315 0.002669602 0.002698914    0
>
> sum(J * rawProbsVec)
[1] 7.26814e-06
>
> sum(Jopt * rawProbsVec)
[1] 7.253857e-06
>
> varJ / (varJ - covJN %*% InvCovNN %*% covNJ)
         [,1]
[1,] 1.031868
> varJopt <- varJ - covJN %*% InvCovNN %*% covNJ
> (varJ - covJN %*% InvCovNN %*% covNJ) / varJ
         [,1]
[1,] 0.969116
> sqrt( (varJ - covJN %*% InvCovNN %*% covNJ) / varJ )
          [,1]
[1,] 0.9844369
>
> pnorm(sum(J * rawProbsVec) / sqrt(varJ), lower.tail = FALSE, log = TRUE)/log(10)
          [,1]
[1,] -33.29985
> pnorm(sum(Jopt * rawProbsVec) / sqrt(varJopt), lower.tail = FALSE, log = TRUE)/log(10)
          [,1]
[1,] -34.19072
>
> S <- c(c(1, -1, -1 ,1), c(1, -1, -1 , 1), c(1, -1, -1, 1), - c(1, -1, -1, 1))
> names(S) <- VecNames
> sum(S * rawProbsVec)
[1] 2.000028
>
> varS <- t(S) %*% Cov %*% S
> covNN <- t(NS) %*% Cov %*% NS
> covSN <- t(S) %*% Cov %*% NS
> covNS <- t(covSN)
>
> varCHSH <- varS
>
> varCHSHopt <- varS - covSN %*% InvCovNN %*% covNS
> varS
             [,1]
[1,] 1.078084e-11
> sqrt(varCHSH / varCHSHopt)
         [,1]
[1,] 1.017637
> covSN %*% solve(covNN)
           NSa1       NSa2      NSb1       NSb2
[1,] -0.4180681 -1.782525 -0.593574 -1.720693
> Sopt <- S - covSN %*% InvCovNN %*% t(NS)
> Sopt
         11dd        11dn      11nd 11nn     12dd       12dn        12nd 12nn     21dd       21dn      21nd 21nn      22dd
[1,] 1.045526 -0.9857173 -0.968757    1 1.050316 -1.014283 -0.9354014    1 1.059105 -0.909652 -1.031243    1 -1.154947
         22dn      22nd 22nn
[1,] 0.909652 0.9354014   -1
>
> CHSH <- sum(S * rawProbsVec)
> CHSH
[1] 2.000028
>
> CHSHopt <- sum(Sopt * rawProbsVec)
> CHSHopt
[1] 2.000028
>
> pnorm((CHSH - 2)/ sqrt(varCHSH), lower.tail = FALSE, log = TRUE)/log(10)
          [,1]
[1,] -17.26464
> pnorm((CHSHopt - 2)/ sqrt(varCHSHopt), lower.tail = FALSE, log = TRUE)/log(10)
          [,1]
[1,] -17.79135
>
>
```