Statistical thinking in “the experiment of the century”

- “If you need statistics, you did the wrong experiment” – Ernest Rutherford

Ernest Rutherford, 1st Baron Rutherford of Nelson, OM, FRS (30 August 1871 – 19 October 1937) was a New Zealand-born British physicist who became known as the father of nuclear physics. Encyclopædia Britannica considers him to be the greatest experimentalist since Michael Faraday

Towards a definitive and successful Bell-type experiment

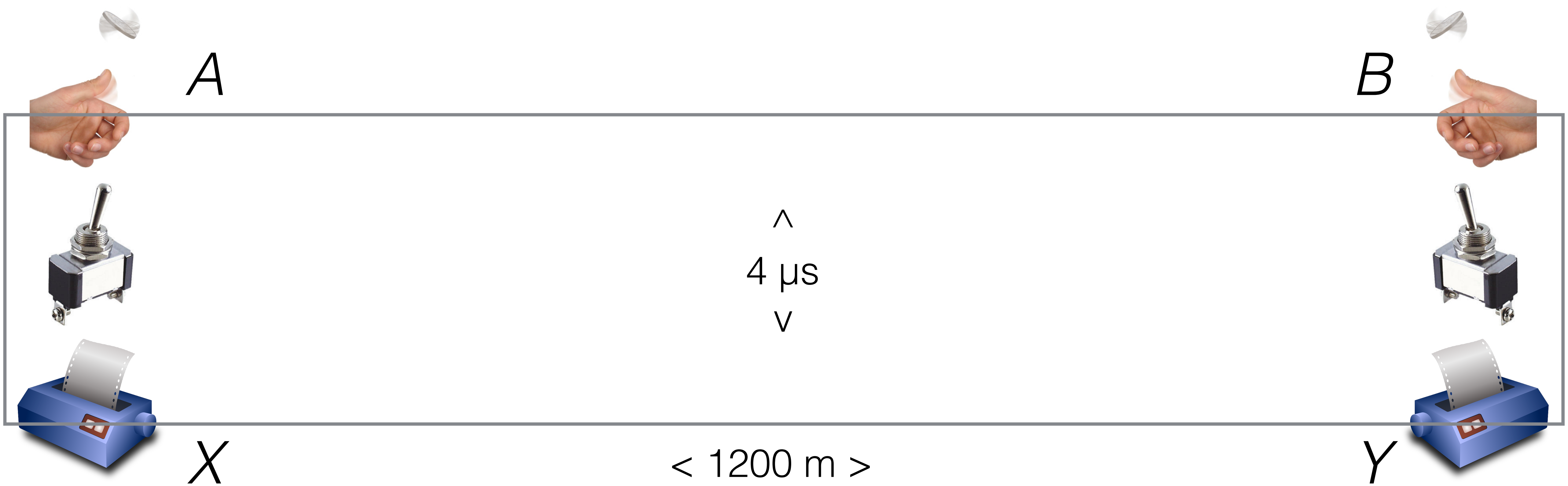
Bell (1964): quantum mechanics (QM) is incompatible
local hidden variables aka local realism
(**locality + realism**)
(provided we assume **freedom / no conspiracy**)

Experimental evidence: …, Aspect (1982), Weihs (1998), Giustina (2013), Christensen (2013) … but not good enough!

Why? technological limitations → loopholes → statistical issues familiar in epidemiology!

time (4 μs)

space (4000 ft)



setting A → outcome X

outcome Y ← setting B

Alice

Bob

speed of light = 1 foot per nanosecond

What is this QM?

- QM models the probability distribution of measurement outcomes, it does not tell us what actually happens
- Measurement outcomes - orthogonal subspaces of complex Hilbert space
- Quantum states - unit vectors in Hilbert space
- Composite systems, measurements, modelled by tensor product
- The probability of a particular outcome = squared length of projection of state vector into corresponding subspace

Pythagoras!

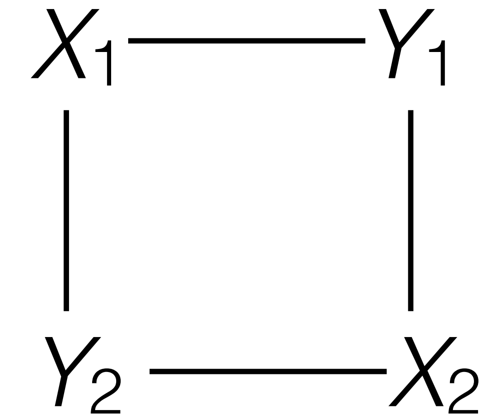
Bell's inequality, Bell's theorem

- Settings A, B take values in $\{1, 2\}$
- Counterfactual outcomes X_{11}, X_{12}, X_{21}, X_{22}, Y_{11}, Y_{12}, Y_{21}, Y_{22} take values in $\{-1, +1\}$
- Actual outcomes $X = X_{AB}$, $Y = Y_{AB}$
- **Freedom (no conspiracy)** $(A, B) \perp\!\!\!\perp (X_{11}, X_{12}, X_{21}, X_{22}, Y_{11}, Y_{12}, Y_{21}, Y_{22})$

Locality, realism, freedom

- **<u>Realism</u>** = existence of counterfactual outcomes
- **<u>Locality</u>** = Alice's outcomes don't depend on Bob's settings and vice-versa
    - $X_1 := X_{11} = X_{12}$, $X_2 := X_{21} = X_{22}$
    - $Y_1 := Y_{11} = Y_{21}$, $Y_2 := Y_{12} = Y_{22}$
    - $X = X_A$, $Y = Y_B$
- **<u>Freedom</u>** = statistical independence of actual settings from counterfactual outcomes
    - $(A, B) \perp\!\!\!\perp (X_1, X_2, Y_1, Y_2)$

Bell's inequality, Bell's theorem



$$X_1 = Y_2 \quad \& \quad Y_2 = X_2 \quad \& \quad X_2 = Y_1 \quad \Rightarrow \quad X_1 = Y_1$$

$$\therefore \quad \boxed{X_1 \neq Y_1 \quad \Rightarrow \quad X_1 \neq Y_2 \quad \text{or} \quad Y_2 \neq X_2 \quad \text{or} \quad X_2 \neq Y_1}$$

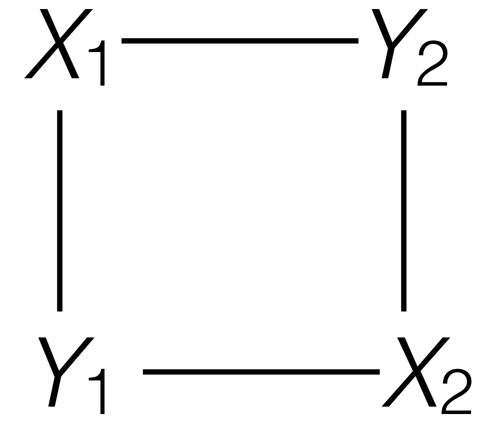
$$\therefore \quad P(X_1 \neq Y_1) \leq P(X_1 \neq Y_2) + P(Y_2 \neq X_2) + P(X_2 \neq Y_1)$$

$$\therefore \quad \boxed{P_{11}(X \neq Y) \leq P_{12}(X \neq Y) + P_{22}(Y \neq X) + P_{21}(X \neq Y)}$$

where $P_{ab}(\ldots) = P(\ldots \mid A = a, B = b)$

NB:(probabilistic) Bell inequality is actually just a simple corollary of a logical implication

Bell's inequality, Bell's theorem



$$P_{11}(X \neq Y) \leq P_{12}(X \neq Y) + P_{22}(Y \neq X) + P_{21}(X \neq Y)$$

For instance: $0.75 \leq 0.25 + 0.25 + 0.25$

But QM promises we can get (approx) 0.85, 0.15, 0.15, 0.15

Notice $E_{ab}(XY) = P_{ab}(X = Y) - P_{ab}(X \neq Y) = 1 - 2\,P_{ab}(X \neq Y)$

Define $S := E_{12}(XY) + E_{22}(XY) + E_{21}(XY) - E_{11}(XY) =: \rho_{12} + \rho_{22} + \rho_{21} - \rho_{11}$

Equivalently: $S \leq 2$ (CHSH: Clauser-Horne-Shimony-Holt, 1969)

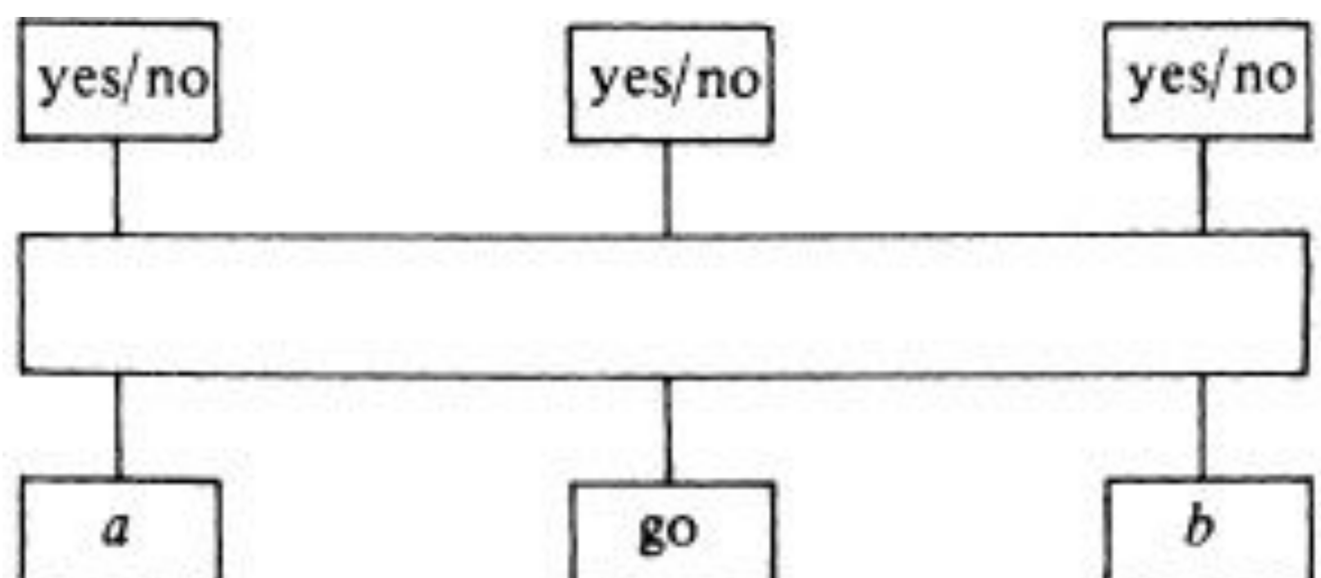
QM promises we can (maximally) get $2\sqrt{2}$ (why not 4 ??)

Bell, 1964 Tsirelson, 1980 Pawlowski, 2009

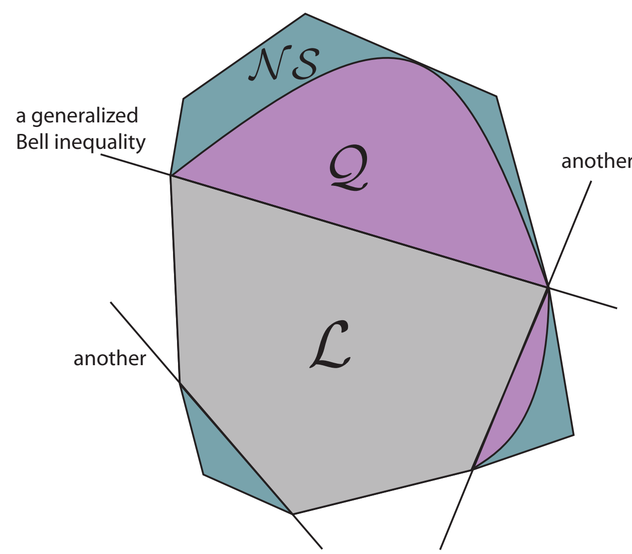
John Bell explains: “I cannot say that action at a distance is required in physics. But I cannot say that you can get away with no action at a distance. You cannot separate off what happens in one place with what happens at another”

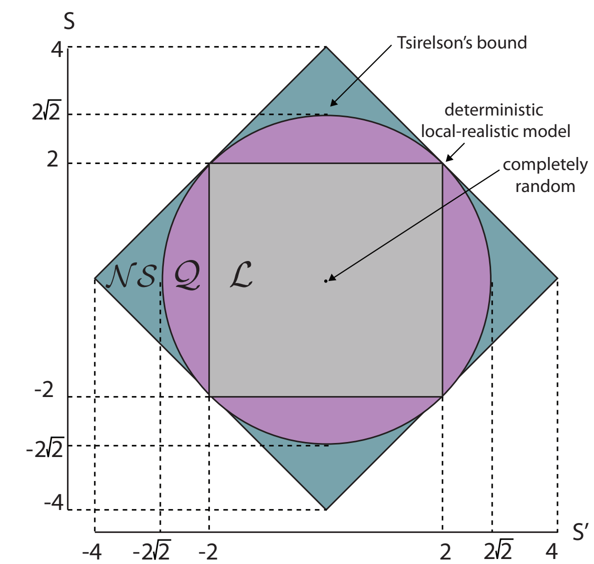
https://www.youtube.com/watch?v=V8CCfOD1iu8

Bell (1981) Bertlmann’s socks and the nature of reality



The local polytope, quantum convex body, no-signalling polytope





<u>Definitions:</u>

$$p_{x,y,a,b} = p(x,\ y \mid a,\ b) = \mathrm{P}(X = x,\ Y = y \mid A = a,\ B = b)$$

$$= \mathrm{P}(X_a = x,\ Y_b = y) \quad \text{under locality + realism + freedom}$$

$$\mathbf{p} = (\ p(x,\ y \mid a,\ b)_{x,y=\pm 1; a,b=1,2}\ ) \in \mathbb{R}^{16}$$

The local-realism polytope, quantum convex body, no-signalling polytope

$p_{x,y,a,b} = p(x,\ y \mid a,\ b) = \mathrm{P}(X = x,\ Y = y \mid A = a,\ B = b)$

$\mathbf{p} = (\ p_{x,y,a,b} : x,\ y = \pm 1;\ a,\ b = 1,2\ ) \in \mathbb{R}^{16}$

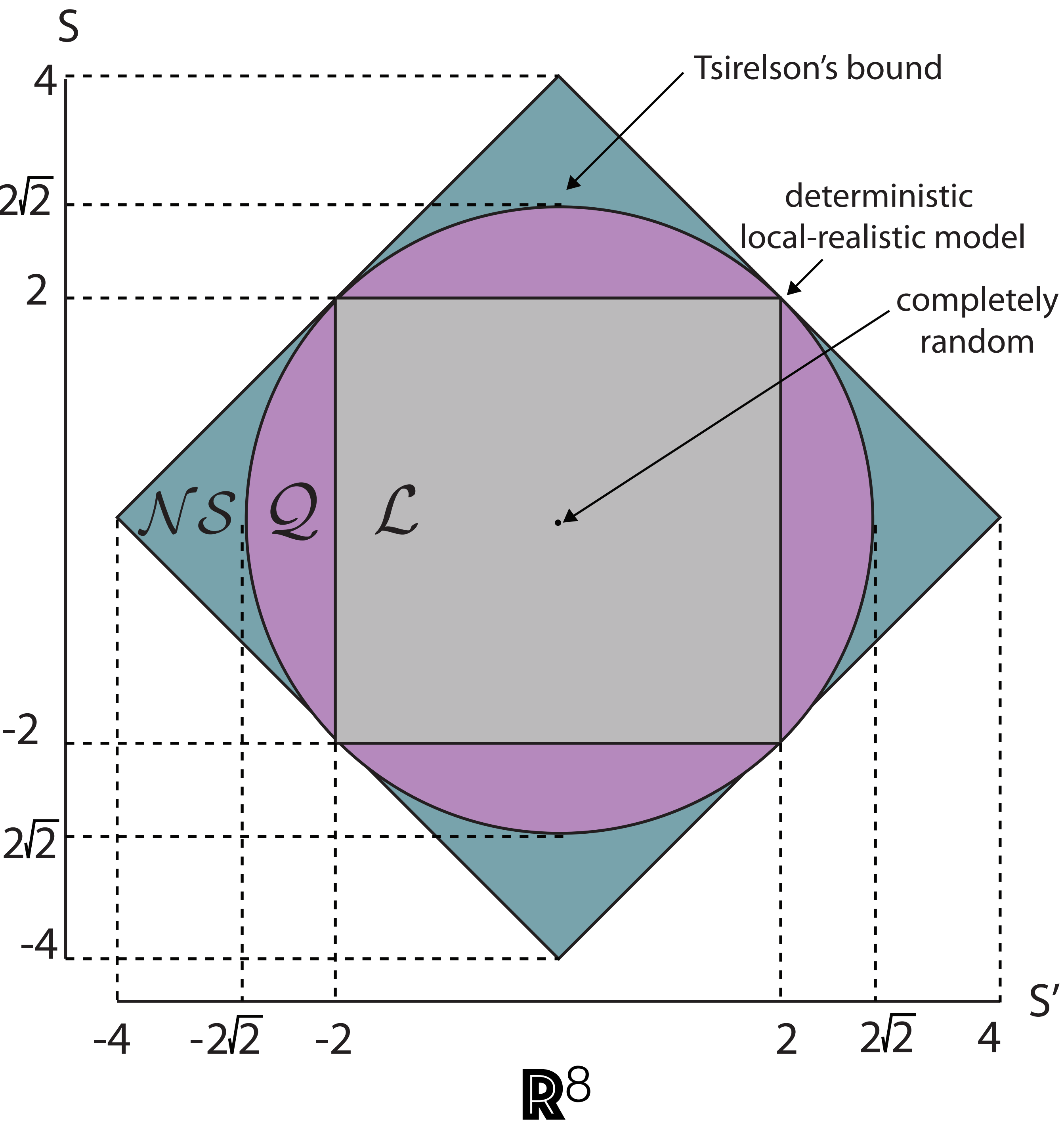
Nonnegativity inequalities: $\forall_{x,y,a,b}\ p_{x,y,a,b} \geq 0$

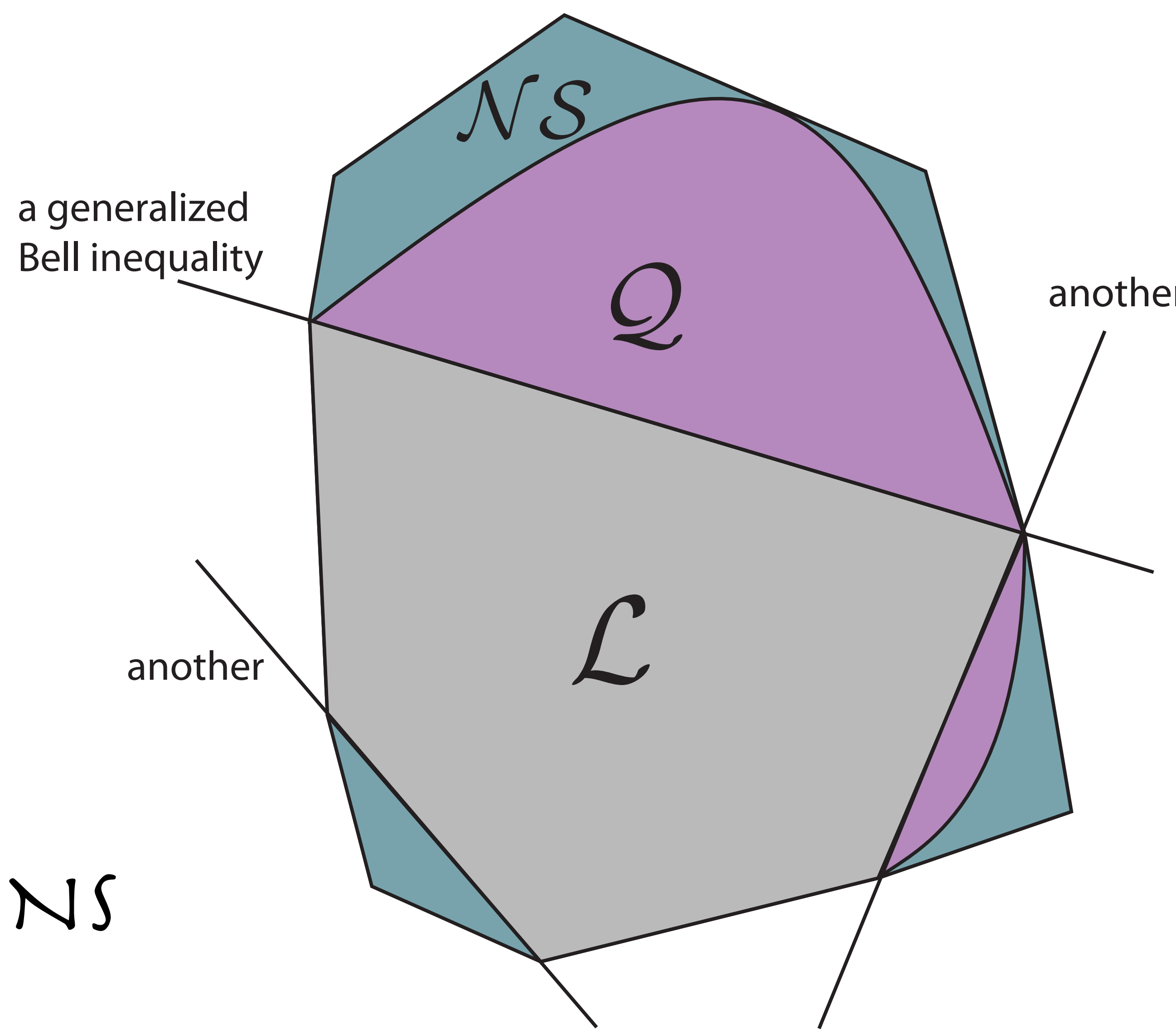
Normalisation equalities: $\forall_{a,b} \sum_{x,y} p_{x,y,a,b} = 1$

No-signalling equalities: $\forall_{a} \sum_{y} p_{x,y,a,b}$ same for all b,
$\forall_{b} \sum_{x} p_{x,y,a,b}$ same for all a

NB: *no-signalling* is a property of all decent physical models. Not just LHV theories …

Caricatures of the local-realism, quantum, no-signalling convex bodies





$$\mathcal{L} \subsetneq \mathcal{Q} \subsetneq \mathcal{NS}$$

Generalised Bell inequalities: *p* parties, *q* settings, *r* outcomes

Left 2 x 2 x 2, right *p* x *q* x *r*

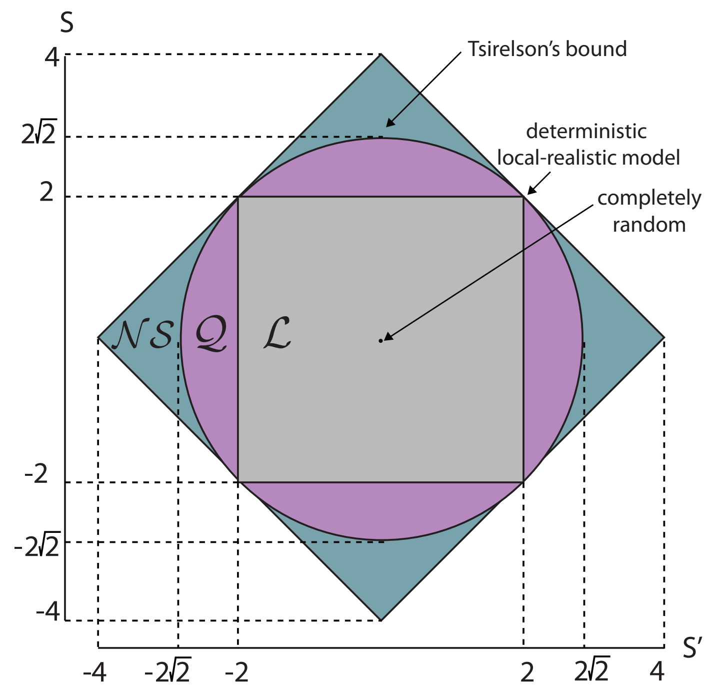
Local-realism extreme point S = ± 2

1	0	0	1
0	0	0	0
1	0	0	1
0	0	0	0

Local-realism S = ± 2 nr QM best

0,375	0,125	0,375	0,125
0,125	0,375	0,125	0,375
0,375	0,125	0,125	0,375
0,125	0,375	0,375	0,125

2x2x2 case



Quantum extreme point S = ± 2√2

0,43	0,07	0,43	0,07
0,07	0,43	0,07	0,43
0,43	0,07	0,07	0,43
0,07	0,43	0,43	0,07

No-signalling extreme point S = ± 4

0,5	0,0	0,5	0,0
0,0	0,5	0,0	0,5
0,5	0,0	0,0	0,5
0,0	0,5	0,5	0,0

Example

Froissart (1981) “I_{3322}”

IL NUOVO CIMENTO Vol. 64 B, N. 2 11 Agosto 1981

Constructive Generalization of Bell’s Inequalities (*).

M. Froissart

Collège de France, Laboratoire de Physique Corpusculaire
11 *Place Marcelin Berthelot*, 75231 *Paris CEDEX* 05 (*France*)

(ricevuto il 23 Marzo 1981)

Summary. — A process to construct all generalizations of Bell’s inequalities in a given experimental situation is presented. In view of the completeness thus obtained, we address a criticism to a very wide class of experiments which purport to rule out locally causal theories by detecting a violation of Bell’s inequalities.

For 3 settings on each side, there are 21 independent equations, 36 inequalities of type (17), 72 of type (20) and 576 of the following type:

$$(21)\qquad \operatorname{Tr} P \begin{pmatrix} 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \geqslant 0 (\times 576) \qquad [f = 1.25]\,.$$

PHYSICAL REVIEW A **82**, 022116 (2010)

Maximal violation of a bipartite three-setting, two-outcome Bell inequality using infinite-dimensional quantum systems

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(Received 21 June 2010; published 26 August 2010)

The I_{3322} inequality is the simplest bipartite two-outcome Bell inequality beyond the Clauser-Horne-Shimony-Holt (CHSH) inequality, consisting of three two-outcome measurements per party. In the case of the CHSH inequality the maximal quantum violation can already be attained with local two-dimensional quantum systems; however, there is no such evidence for the I_{3322} inequality. In this paper a family of measurement operators and states is given which enables us to attain the maximum quantum value in an infinite-dimensional Hilbert space. Further, it is conjectured that our construction is optimal in the sense that measuring finite-dimensional quantum systems is not enough to achieve the true quantum maximum. We also describe an efficient iterative algorithm for computing quantum maximum of an arbitrary two-outcome Bell inequality in any given Hilbert space dimension. This algorithm played a key role in obtaining our results for the I_{3322} inequality, and we also applied it to improve on our previous results concerning the maximum quantum violation of several bipartite two-outcome Bell inequalities with up to five settings per party.

DOI: 10.1103/PhysRevA.82.022116 PACS number(s): 03.65.Ud, 03.67.−a

However, there were still a few exceptions, where the upper bound value resulting from the NPA method [10] did not match the best lower bound result. The most interesting one was the case of I_{3322}. This is the smallest case we considered, and perhaps the simplest tight Bell inequality after the CHSH one, with only three measurement settings per party. It was introduced by Froissart [21] back in 1981, and recently reinvented in Refs. [22, 23]. It reads

$$\begin{aligned} I_{3322} \equiv & -\langle A_2\rangle - \langle B_1\rangle - 2\langle B_2\rangle \\ & + \langle A_1B_1\rangle + \langle A_1B_2\rangle + \langle A_2B_1\rangle + \langle A_2B_2\rangle \\ & - \langle A_1B_3\rangle + \langle A_2B_3\rangle - \langle A_3B_1\rangle + \langle A_3B_2\rangle \le 0. \quad (3) \end{aligned}$$

In a local classical model we have the maximum value of 0, while the largest violation one could get with qubits was 0.25, which could already be achieved with a maximally entangled pair of qubits (see e.g., [11, 13, 16–18, 23]). On the other hand, the best upper bounds are based on the NPA method [10] and at level three it yields the significantly higher upper bound, 0.250 875 56 [11, 16]. We could even go above level three to an intermediate level in [13], and presently we have got the upper bound 0.250 875 38 at level four. From the dependence

Experiment

- The definitive experiment has now been done
- It was done first, in Delft (and soon after in at least three other places)

Delft:
the Hanson diamond group

doi:10.1038/nature12016

Heralded entanglement between solid-state qubits separated by three metres

H. Bernien[1], B. Hensen[1], W. Pfaff[1], G. Koolstra[1], M. S. Blok[1], L. Robledo[1], T. H. Taminiau[1], M. Markham[2], D. J. Twitchen[2], L. Childress[3] & R. Hanson[1]

86 | NATURE | VOL 497 | 2 MAY 2013

Unconditional quantum teleportation between distant solid-state quantum bits

W. Pfaff,[1*] B. J. Hensen,[1] H. Bernien,[1] S. B. van Dam,[1] M. S. Blok,[1] T. H. Taminiau,[1] M. J. Tiggelman,[1] R. N. Schouten,[1] M. Markham,[2] D. J. Twitchen,[2] R. Hanson[1]†

Realizing robust quantum information transfer between long-lived qubit registers is a key challenge for quantum information science and technology. Here we demonstrate unconditional teleportation of arbitrary quantum states between diamond spin qubits separated by 3 meters. We prepare the teleporter through photon-mediated heralded entanglement between two distant electron spins and subsequently encode the source qubit in a single nuclear spin. By realizing a fully deterministic Bell-state measurement combined with real-time feed-forward, quantum teleportation is achieved upon each attempt with an average state fidelity exceeding the classical limit. These results establish diamond spin qubits as a prime candidate for the realization of quantum networks for quantum communication and network-based quantum computing.

532 1 AUGUST 2014 • VOL 345 ISSUE 6196 sciencemag.org **SCIENCE**

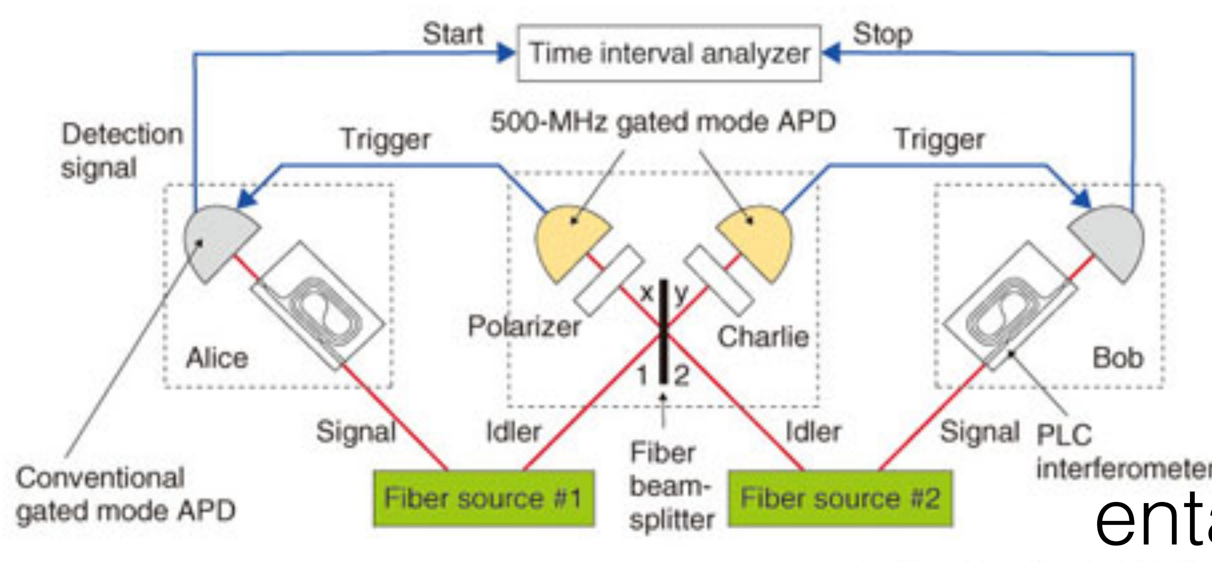
Delft: the Hanson diamond group

http://www.tudelft.nl/en/current/latest-news/article/detail/beam-me-up-data/

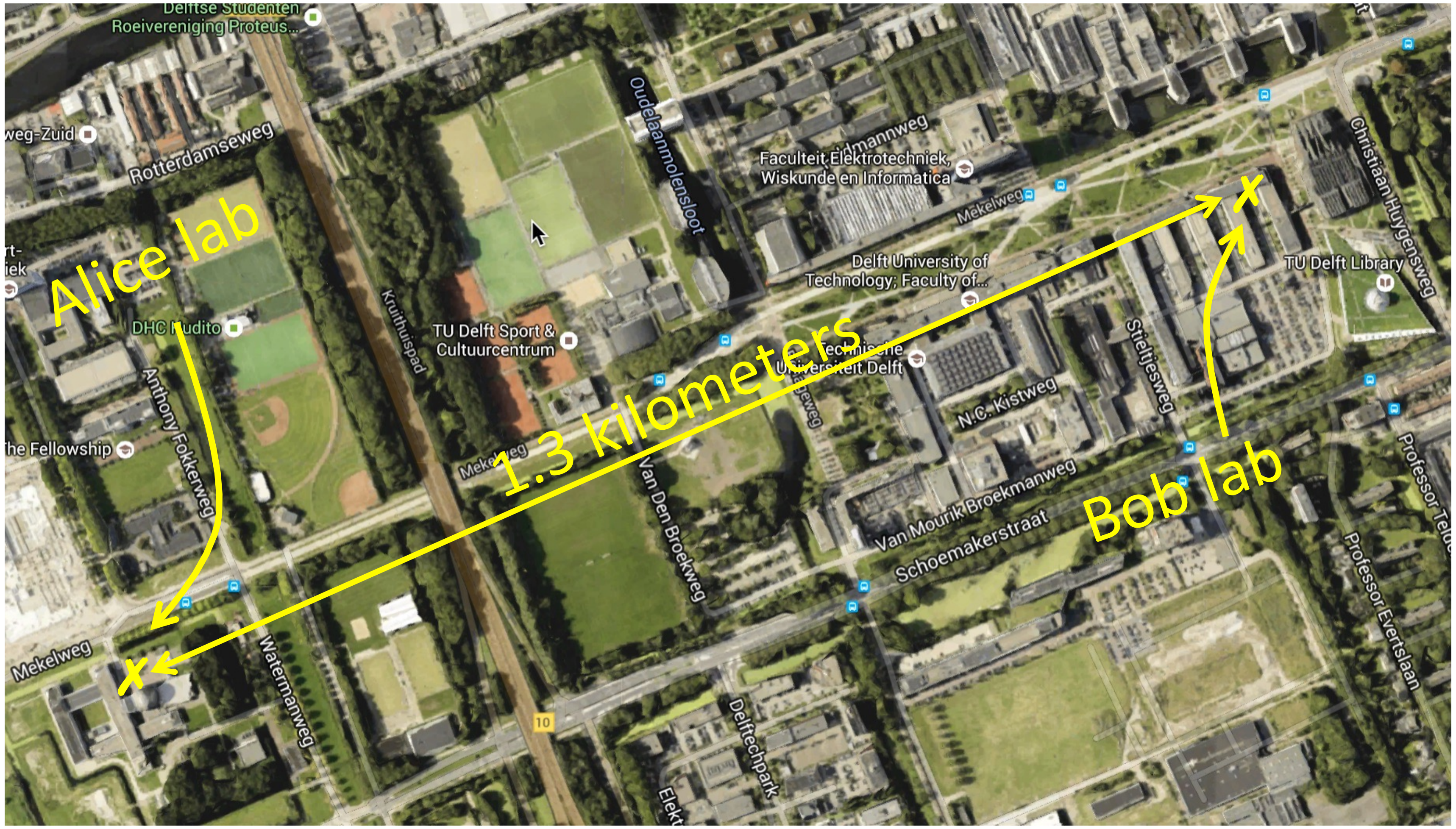
Diamonds
Hanson's research group produces qubits using electrons in diamonds. ‘We use diamonds because ‘mini prisons’ for electrons are formed in this material whenever a nitrogen atom is located in the position of one of the carbon atoms. The fact that we're able to view these miniature prisons individually makes it possible for us to study and verify an individual electron and even a single atomic nucleus. We're able to set the spin (rotational direction) of these particles in a predetermined state, verify this spin and subsequently read out the data. We do all this in a material that can be used to make chips out of. This is important as many believe that only chip-based systems can be scaled up to a practical technology,’ explains Hanson.

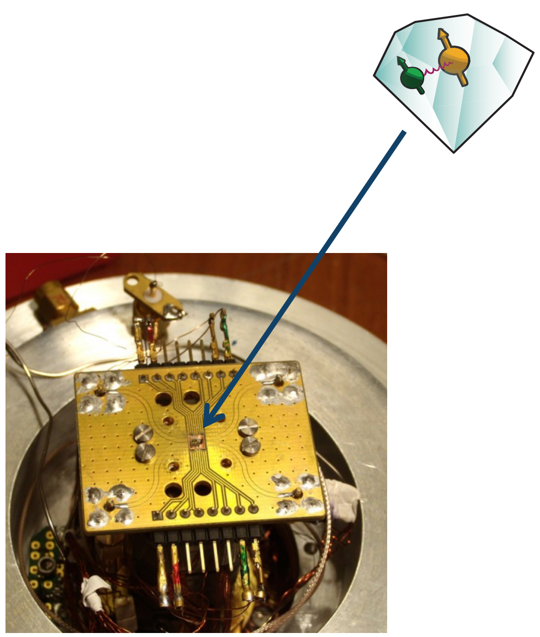
Holy Grail
Hanson is planning to repeat the experiment this summer over a distance of 1300 metres, with chips located in various buildings on TU Delft's campus. This experiment could be the first that meets the criteria of the 'loophole-free Bell test', and could provide the ultimate evidence to disprove Einstein’s rejection of entanglement. Various research groups, including Hanson's, are currently striving to be the first to realise a loophole-free Bell test, which is considered 'Holy Grail' within quantum mechanics.

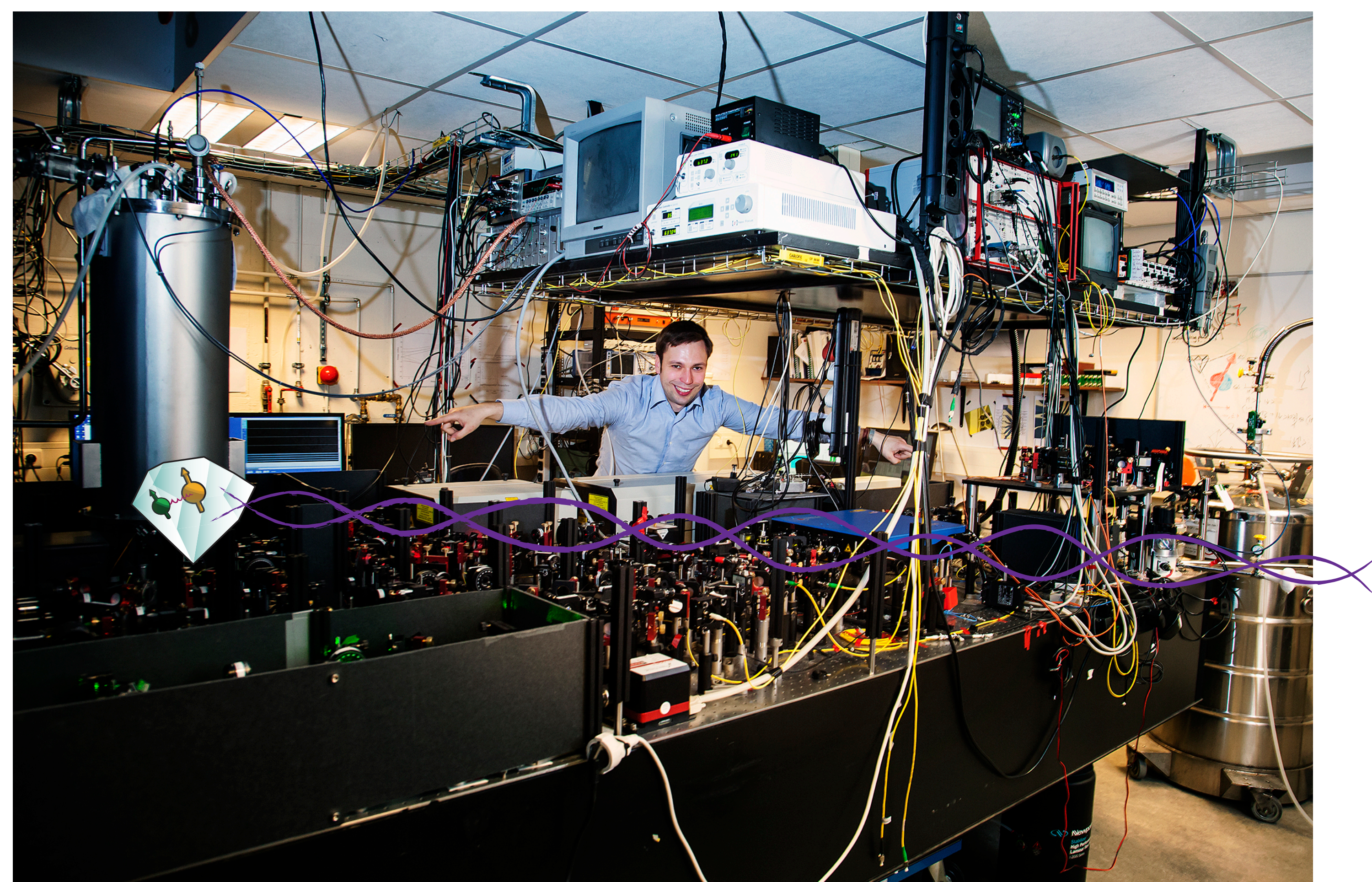
29 May 2014



entanglement swapping


Alice lab
1.3 kilometers
Bob lab
Delftse Studenten Roeivereniging Proteus...
Rotterdamseweg
DHC Hudito
TU Delft Sport & Cultuurcentrum
Faculteit Elektrotechniek, Wiskunde en Informatica
Mekelweg
Delft University of Technology: Faculty of...
Technische Universiteit Delft
N.C. Kistweg
Van Mourik Broekmanweg
Schoemakerstraat
TU Delft Library
Stieltjesweg
Christiaan Huygensweg
Professor Evertslaan
Oudelaanmolensloot
Kruithuispad
Van Den Broekweg
Anthony Fokkerweg
The Fellowship
Watermanweg
Delftechpark





Statistical issues

- Detection loophole – intention to treat principle!
- Memory loophole – randomisation and martingale theory!
- Time variation – randomisation and martingale theory!
- Statistical significance: based on randomisation of settings, not on an assumed physical model (no "iid" assumption)
- Time variation – turn a bug into a feature by dynamic optimisation of test statistic
- Statistical optimisation – from "$S \leq 2$" to

$$S + \sum_{\text{no-signalling equalities}} c_{\text{n.s.e.}}\, \text{n.s.e.} \leq 2$$

http://rpubs.com/gill1109

Turning around the randomness: Bell inequalities are logical inequalities

- Suppose A, B are independent fair Bernoulli
- Define $I_{ab} = I(A = a, B = b)$
- Condition on values of X_1, X_2, Y_1, Y_2 and define $\delta_{ab} = I(x_a \neq y_b)$
- Observe that
$E(\delta_{11}\ I_{11} - \delta_{12}\ I_{12} - \delta_{22}\ I_{22} - \delta_{21}\ I_{21}) \leq 0$ because
$x_1 \neq y_1 \quad \Rightarrow \quad x_1 \neq y_2$ or $y_2 \neq x_2$ or $x_2 \neq y_1$

This leads to martingale tests: protection against: time dependence, trends and jumps, opportunistic stopping or skipping

Want to know more?

- http://www.slideshare.net/gill1109/epidemiology-meets-quantum-statistics-causality-and-bells-theorem

- http://www.math.leidenuniv.nl/~gill

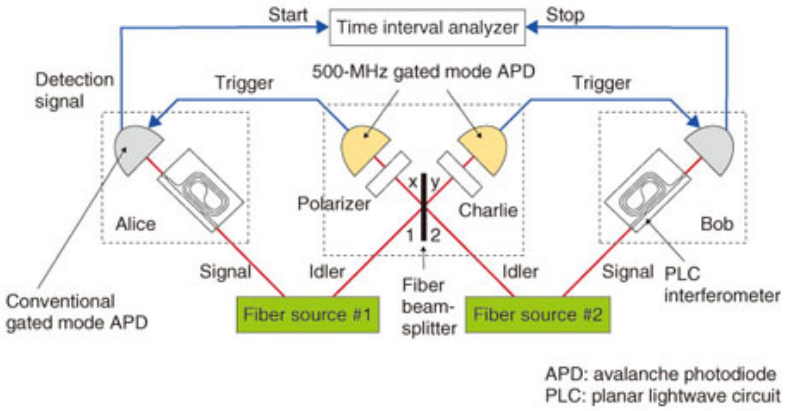
- Survey paper in *Statistical Science*

Statistical Science
2014, Vol. 29, No. 4, 512–528
DOI: 10.1214/14-STS490

Statistics, Causality and Bell's Theorem

Richard D. Gill

Abstract. Bell's [*Physics* **1** (1964) 195–200] theorem is popularly supposed to establish the nonlocality of quantum physics. Violation of Bell's inequality in experiments such as that of Aspect, Dalibard and Roger [*Phys. Rev. Lett.* **49** (1982) 1804–1807] provides empirical proof of nonlocality in the real world. This paper reviews recent work on Bell's theorem, linking it to issues in causality as understood by statisticians. The paper starts with a proof of a strong, finite sample, version of Bell's inequality and thereby also of Bell's theorem, which states that quantum theory is incompatible with the conjunction of three formerly uncontroversial physical principles, here referred to as *locality*, *realism* and *freedom*.



Postscript

I cannot say that action at a distance is required in physics. But I cannot say that you can get away with no action at a distance. You cannot separate off what happens in one place with what happens at another – John Bell

https://www.youtube.com/watch?v=V8CCfOD1iu8

Nature produces chance events (irreducibly chance-like!) which can occur at widely removed spatial locations without anything propagating from point to point along any path joining those locations. … The chance-like character of these effects prevents any possibility of using this form of non locality to communicate, thereby saving from contradiction one of the fundamental principles of relativity theory according to which no communication can travel faster than the speed of light – Nicolas Gisin

Quantum Chance: Nonlocality, Teleportation and Other Quantum Marvels. Springer, 2014