

Read chapter 1.1 and chapter 1.2 . Answer the questions below. The test on Thursday will be very similar to these questions. If you get stuck, you can ask for help during the werkcollege before the test.

- 4.** Find the point of intersection of the lines $x_1 + 2x_2 = -13$ and $3x_1 - 2x_2 = 1$

Determine if the systems in Exercises 15 and 16 are consistent. Do not completely solve the systems.

15.

$$\begin{aligned}x_1 - 6x_2 &= 5 \\x_2 - 4x_3 + x_4 &= 0 \\-x_1 + 6x_2 + x_3 + 5x_4 &= 3 \\-x_2 + 5x_3 + 4x_4 &= 0\end{aligned}$$

- 18.** Do the three planes $2x_1 + 4x_2 + 4x_3 = 4$, $x_2 - 2x_3 = -2$, and $2x_1 + 3x_2 = 0$ have at least one common point of intersection? Explain.

In Exercises 23 and 24, key statements from this section are either quoted directly, restated slightly (but still true), or altered in some way that makes them false in some cases. Mark each statement True or False, and *justify* your answer. (If true, give the approximate location where a similar statement appears, or refer to a definition or theorem. If false, give the location of a statement

- 24.**
- Two matrices are row equivalent if they have the same number of rows.
 - Elementary row operations on an augmented matrix never change the solution set of the associated linear system.
 - Two equivalent linear systems can have different solution sets.
 - A consistent system of linear equations has one or more solutions.

In Exercises 1 and 2, determine which matrices are in reduced echelon form and which others are only in echelon form.

1. a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

Row reduce the matrices in Exercises 3 and 4 to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

3. $\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$

Find the general solutions of the systems whose augmented matrices are given in Exercises 7–14.

7. $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$

8. $\begin{bmatrix} 1 & -3 & 0 & -5 \\ -3 & 7 & 0 & 9 \end{bmatrix}$

9. $\begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & -3 & 4 & -6 \end{bmatrix}$

10. $\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$

22. a. The reduced echelon form of a matrix is unique.
b. If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.
c. The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.
d. A general solution of a system is an explicit description of all solutions of the system.
e. Whenever a system has free variables, the solution set contains many solutions.
23. Suppose the coefficient matrix of a linear system of four equations in four variables has a pivot in each column. Explain why the system has a unique solution.