Week 1 Solutions

5) Yes $at^2 + bt^2 = (a+b)t^2$, $c(at^2) = (ca)t^2$, $0.t^2 = 0$.

6) No, 0 is not of this form.

7) No, $1 \in V$ but $\frac{1}{2} \cdot 1 \notin V$, not closed under scalar mult.

8) Yes, this is the kernel of the linear map to $\mathbb{R}$ given by 'evaluate at 0' (or check axioms)

9) $H = \text{Span}\left\{\begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}\right\}$, hence is a subspace.

13. a) No; only contains 3 vectors.

b) Infinitely many.

c) Equivalent; does $a_1\underline{v_1} + a_2\underline{v_2} + a_3\underline{v_3} = \underline{w}$ have a solution?

Row reduce $A(\vec{w})$: $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(not reduced echelon form - doesn't matter)
Last column NOT pivot, so there is a solution.

So YES.

3) Rowred: $\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & -2 & 4 \\ 0 & 1 & 3 & -2 \end{bmatrix}$,

so gen sol'n

$x_1 = 2x_3 - 4x_4$
$x_2 = -3x_3 + 2x_4$
$x_3 = 1x_3 + 0x_4$
$x_4 = 0x_3 + 1x_4$

$\underline{x} = \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix} t$, $s, t \in \mathbb{R}$.

$\text{Nul}(A) = \text{Span}\left\{\begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix}\right\}$.

7) Not a vector ~~set~~ space; does not contain $\underline{0}$

8) Same as 7.

35) 1) $\underline{0} \in T(U)$: well, $\underline{0} \in U$ and $T(\underline{0}) = \underline{0}$.

2) $T(U)$ closed under $+$:

let $\underline{v}_1, \underline{v}_2 \in T(U)$, & choose $\underline{u}_1, \underline{u}_2$

with $T(\underline{u}_i) = \underline{v}_i$, then $T(\underline{u}_1 + \underline{u}_2) = T(\underline{u}_1) + T(\underline{u}_2)$

$= \underline{v}_1 + \underline{v}_2$

is in $T(U)$

3) $T(U)$ closed under scalar mult:

let $c \in \mathbb{R}$, $\underline{v} \in T(U)$, Say $\underline{v} = T(\underline{u})$,

then $c\underline{v} = cT(\underline{u}) = T(c\underline{u}) \in T(U)$.