# Homework Solutions

1) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ $\quad \underline{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix}$$

$$A^T \underline{b} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \end{bmatrix}$$

ACM:

$$\left[\begin{array}{cc:c} 4 & 6 & 6 \\ 6 & 14 & 11 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc:c} 2 & 3 & 3 \\ 6 & 14 & 11 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc:c} 2 & 3 & 3 \\ 0 & 5 & 2 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cc:c} 1 & 3/2 & 3/2 \\ 0 & 1 & 2/5 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc:c} 1 & 0 & 9/10 \\ 0 & 1 & 2/5 \end{array}\right]$$

$$y = \frac{9}{10} + \frac{2}{5} x$$

2) $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \\ 1 & 5 \end{bmatrix}$ $\quad \underline{b} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 12 & 46 \end{bmatrix} \qquad A^T \underline{b} = \begin{bmatrix} 6 \\ 25 \end{bmatrix}$$

ACM $\left[\begin{array}{cc|c} 4 & 12 & 6 \\ 12 & 46 & 25 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|c} 4 & 12 & 6 \\ 0 & 10 & 7 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc} 1 & 3 & 3/2 \\ 0 & 1 & 7/10 \end{array}\right]$

$\rightsquigarrow \begin{bmatrix} 1 & 0 & -6/10 \\ 0 & 1 & 7/10 \end{bmatrix}$ $\qquad y = -\frac{6}{10} + \frac{7}{10}x.$

5) ~~We apply theorem: The columns of A are linearly independent iff the least squares solution is unique.~~

5) We apply theorem 14: The columns of A are linearly independent if & only if the LSS is unique.

The matrix A is given by

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$$

So the colums are linearly independent if & only if there are at least two distinct $x_i$.

7) $A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 3 & 9 \\ 4 & 16 \\ 5 & 25 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 1.8 \\ 2.7 \\ 3.4 \\ 3.8 \\ 3.9 \end{bmatrix}.$