

LAI HW1 Sol'n's

①

4

$$\begin{aligned} x_1 + 2x_2 &= -13 \\ 3x_1 - 2x_2 &= 1 \end{aligned} \quad \left[\begin{array}{cc|c} 1 & 2 & -13 \\ 3 & -2 & 1 \end{array} \right] \xrightarrow{r_2 \rightarrow r_2 - 3r_1} \left[\begin{array}{cc|c} 1 & 2 & -13 \\ 4 & -6 & -38 \end{array} \right]$$

$$r_2 \rightarrow r_2/4 \rightarrow \left[\begin{array}{cc|c} 1 & 2 & -13 \\ 1 & 0 & -3 \end{array} \right] \xrightarrow{r_1 \rightarrow r_1 - r_2} \left[\begin{array}{cc|c} 1 & 2 & -13 \\ 0 & -2 & 10 \end{array} \right]$$

$$\begin{aligned} r_1 &\rightarrow r_1 + r_2 \\ r_2 &\rightarrow r_2/2 \end{aligned} \rightarrow \left[\begin{array}{cc|c} 2 & 0 & -3 \\ 0 & 1 & -5 \end{array} \right] \quad \begin{aligned} x_1 &= -3 \\ x_2 &= -5 \end{aligned}$$

15 ACM: $\left[\begin{array}{cccc|c} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ -1 & 6 & 1 & 5 & 3 \\ 0 & -1 & 5 & 4 & 0 \end{array} \right] \xrightarrow{\begin{matrix} r_3 \rightarrow r_3 + r_1 \\ r_4 \rightarrow r_4 + r_2 \end{matrix}} \left[\begin{array}{cccc|c} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 1 & 5 & 0 \end{array} \right]$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & -8 \end{array} \right]$$

last column is a pivot,
inconsistent

18 ACM $\left[\begin{array}{cccc|c} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{array} \right] \xrightarrow{r_3 \rightarrow r_3 - r_1} \left[\begin{array}{cccc|c} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -4 & -4 \end{array} \right]$

$$r_3 \rightarrow r_3 + r_2 \rightarrow \left[\begin{array}{cccc|c} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -6 & -6 \end{array} \right]$$

- last col not pivot

- in consistent

- so there is a point of intersection.

24a) False, e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ both in reduced echelon form.

not = so not equivalent.

b) True,

c) False, see definition of equivalence.

d) True, by definition.

1) a) reduced echelon

b) reduced echelon

c) not in echelon form

d) echelon form, not reduced.

$$3) \begin{bmatrix} \textcircled{1} & 2 & 4 & 8 \\ 2 & 4 & \textcircled{6} & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix} \begin{matrix} r_2/2 \\ r_3/3 \end{matrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} \begin{matrix} r_3 \rightarrow r_3 - r_2 \\ r_2 \rightarrow r_2 - r_1 \end{matrix} \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} r_2 \leftrightarrow -r_2 \\ r_1 \rightarrow r_1 - 4r_2 \\ r_3 \rightarrow r_3 - 2r_2 \end{matrix} \begin{bmatrix} \textcircled{1} & 2 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivot cols 1 8 3

$$4) \begin{bmatrix} \textcircled{1} & 2 & 4 & 5 \\ 2 & \textcircled{4} & 5 & 4 \\ 4 & 5 & \textcircled{4} & 2 \end{bmatrix} \begin{matrix} r_2 \rightarrow r_2 - 2r_1 \\ r_3 \rightarrow r_3 - 4r_1 \end{matrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & -3 & -6 \\ 0 & -3 & -6 & -6 \end{bmatrix} \begin{matrix} r_2 \leftrightarrow r_3 \\ \text{divide by } -3 \end{matrix} \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{matrix} r_1 \rightarrow r_1 - 4r_3 \\ r_2 \rightarrow r_2 - 2r_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} \textcircled{1} & 0 & 0 & 1 \\ 0 & \textcircled{1} & 0 & -2 \\ 0 & 0 & \textcircled{1} & 2 \end{bmatrix}$$

Pivot columns
1, 2, 3