LA1 HW1 Soln.

4 $x_1 + 2x_2 = -13$

$3x_1 - 2x_2 = 1$

$$\begin{bmatrix} 1 & 2 & -13 \\ 3 & -2 & 1 \end{bmatrix} \xrightarrow{r_2 \mapsto r_2 - r_1} \begin{bmatrix} 1 & 2 & -13 \\ 4 & 0 & -12 \end{bmatrix}$$

$$\xrightarrow{r_2 \mapsto r_2/4} \begin{bmatrix} 1 & 2 & -13 \\ 1 & 0 & -3 \end{bmatrix} \xrightarrow{r_2 \mapsto r_2 - r_1} \begin{bmatrix} 1 & 2 & -13 \\ 0 & -2 & 10 \end{bmatrix}$$

$$\xrightarrow[r_2 \mapsto r_2/-2]{r_1 \mapsto r_1 + r_2} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \end{bmatrix}$$

$x_1 = -3$

$x_2 = -5$

15 ACM:

$$\begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ -1 & 6 & 1 & 5 & 3 \\ 0 & -1 & 5 & 4 & 0 \end{bmatrix} \xrightarrow[r_4 \mapsto r_4 + r_2]{r_3 \mapsto r_3 + r_1} \begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 1 & 5 & 0 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & (-8) \end{bmatrix}$$

Last column is a pivot,

so we have an equation 0 = -8, so not consistent.

18 ACM

$$\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix} \xrightarrow{r_3 \mapsto r_3 - r_1} \begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -4 & -4 \end{bmatrix}$$

$$\xrightarrow{r_3 \mapsto r_3 + r_2} \begin{bmatrix} (2) & 4 & 4 & 4 \\ 0 & (1) & -2 & -2 \\ 0 & 0 & (-6) & -6 \end{bmatrix}$$

Re-write in terms of equations.
The last row fixes the value of x_3.
Then the second row fixed the value of x_2
Then the first row fixes the value of x_1.

We will come back to this more next week.

24 a) False, e.g. $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ and $\begin{bmatrix}0&0\\0&0\end{bmatrix}$ both in reduced echelon form.

not = sonst equivalent.

b) True,

c) False, see definition of equivalence

d) True, by definitions

---

1) a) reduced echelon

b) reduced echelon

c) not in echelon form

d) echelon form, not reduced.

3) $\begin{bmatrix}\textcircled{1}&2&4&8\\2&4&\textcircled{6}&8\\3&6&9&12\end{bmatrix} \xrightarrow[r_3/3]{r_2/2} \begin{bmatrix}1&2&4&8\\1&2&3&4\\1&2&3&4\end{bmatrix} \xrightarrow[r_2\mapsto r_2-r_1]{r_3\mapsto r_3-r_2} \begin{bmatrix}1&2&4&8\\0&0&-1&-4\\0&0&0&0\end{bmatrix}$

$\xrightarrow{r_2\mapsto -r_2} \begin{bmatrix}\textcircled{1}&2&0&0\\0&0&\textcircled{1}&4\\0&0&0&0\end{bmatrix}$ Pivot cols 1 & 3

$r_1\mapsto r_1-4r_2$, $r_3\mapsto r_3+r_2$

4) $\begin{bmatrix}\textcircled{1}&2&4&5\\2&\textcircled{4}&5&4\\4&5&\textcircled{4}&2\end{bmatrix} \xrightarrow[r_3\mapsto r_3-2r_1]{r_2\mapsto r_2-2r_1} \begin{bmatrix}1&2&4&5\\0&0&-3&-6\\0&-3&-6&-6\end{bmatrix} \xrightarrow[\text{divide by }3]{r_2\leftrightarrow r_3} \begin{bmatrix}1&2&4&5\\0&1&2&2\\0&0&1&2\end{bmatrix}$

$r_1\mapsto r_1-4r_3$, $r_2\mapsto r_2-2r_3$: $\begin{bmatrix}1&2&0&-3\\0&1&0&-2\\0&0&1&2\end{bmatrix} \xrightarrow{r_1\mapsto r_1-2r_2} \begin{bmatrix}\textcircled{1}&0&0&1\\0&\textcircled{1}&0&-2\\0&0&\textcircled{1}&2\end{bmatrix}$ Pivot columns 1, 2, 3