~~17~~ Row reduction of A is

$$\begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row reduction of B is

$$\begin{bmatrix} 1 & 0 & -11 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

17: 3 rows contain a pivot, so the equation $A\underline{x} = \underline{b}$ is <u>not</u> always consistent (theorem 4)

18. No. Not all rows of B contain a pivot (thm 4 again)
Columns do <u>not</u> span $\mathbb{R}^4$ (they don't lie in $\mathbb{R}^3$)

19. No - therem 4 again.

20. No - ——— " ———.

15 ACM $\left[\begin{array}{cc|c} 3 & -1 & b_1 \\ -9 & 3 & b_2 \end{array}\right] \to \left[\begin{array}{cc|c} 3 & -1 & b_1 \\ 0 & 0 & b_2+3b_1 \end{array}\right]$

Consistent if & only if $b_2+3b_1=0$.
(eg. not for $\underline{b}=\begin{bmatrix}1\\0\end{bmatrix}$)

36 Unique sol'n $\Rightarrow$ no free variable
$\Rightarrow$ pivot in every column of A
$\Rightarrow$ 4 pivots
$\Rightarrow$ pivot in every row of A
$\Rightarrow$ columns of A span $\mathbb{R}^4$
theorem 4.

1 $\left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ -2 & -7 & 1 & 0 \\ 4 & 2 & 7 & 0 \end{array}\right] \xrightarrow{r_3 \mapsto r_3 - r_1 + r_2} \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ -2 & -7 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

So ≤ 2 pivots so ≥ 1 free variable,
so $\exists$ non-trivial solution.

2 3

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ -2 & -3 & -4 & 0 \\ 2 & -4 & 9 & 0 \end{array}\right] \xrightarrow[r_3 \mapsto r_3 - 2r_1]{r_2 \mapsto r_2 + 2r_1} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & -7 & 2 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

Echelon form, 3 pivots, so no non-trivial sol'n.

12 Row reduce to

$$\begin{bmatrix} 1 & -2 & 3 & 0 & 29 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 2x_2 - 3x_3 - 29x_5 \\ x_2 &= 1x_2 + 0x_3 + 0x_5 \\ x_3 &= 0x_2 + 1x_3 + 0x_5 \\ x_4 &= 0x_2 + 0x_3 - 4x_5 \\ x_5 &= 0x_2 + 0x_3 + 1x_5 \\ x_6 &= 0x_2 + 0x_3 + 0x_5 \end{aligned}$$

General sol'n $\underline{x} = s\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + u\begin{bmatrix} -29 \\ 0 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix}$

$(s, t, u \in \mathbb{R})$

28 a) No - ~~one~~ no free variable

b) Yes - ~~[illegible]~~ every row has a pivot

29 a) Yes - one free variable

b) No - $\exists$ row without pivot.

5

1 Yes (row reduced matrix has no free variable)

2 Yes —— " ——.

11 $\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ -2 & -6 & 2 & 0 \\ 4 & 7 & h & 0 \end{array}\right] \overset{\text{R.red}}{\rightsquigarrow} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & h+4 & 0 \end{array}\right]$

linearly indep $\Leftrightarrow$ no free variable

$\Leftrightarrow h+4 \neq 0$

$\Leftrightarrow h \neq -4$.

12 $\left[\begin{array}{ccc|c} 3 & -6 & 9 & 0 \\ -6 & 4 & h & 0 \\ 1 & -3 & 3 & 0 \end{array}\right] \overset{\text{R.red}}{\rightsquigarrow} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & h+18 & 0 \end{array}\right]$

lin. indep $\Leftrightarrow$ no free variable

$\Leftrightarrow h+18 \neq 0$

$\Leftrightarrow h \neq -18$.

22 a. ~~Ya~~ T. $\underline{w} = x_1\underline{u} + x_2\underline{v}$, so $x_1\underline{u} + x_2\underline{v} - \underline{w} = 0$. 6

b. ~~Yes~~ T.

c. F. eg. $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\end{bmatrix}\right\}$ is lin dependent

d. F, eg. $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\end{bmatrix}\right\}$.

40. ~~The~~ The system $A\underline{x} = \underline{b}$ has no free variable, as there are n variables & n basic variables.

2) $T(\underline{u}) = A\underline{u} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ $\quad T(\underline{v}) = A\underline{v} = \begin{bmatrix} a/3 \\ b/3 \\ c/3 \end{bmatrix}$

3) ~~R~~ ACM: $\left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ -3 & 1 & 6 & 3 \\ 2 & -2 & -1 & -1 \end{array}\right] \xrightarrow{\text{R-Red}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{array}\right]$.

~~No~~ Last column not pivot, $\Rightarrow$ consistent
No free variable $\Rightarrow$ solution unique.
As eqns: $x_1 = 7$
$x_2 = 6$
$x_3 = 3$ $\quad$ Sol'n: $\begin{bmatrix} 7 \\ 6 \\ 3 \end{bmatrix}$.

7) $a = 5$, $b = 6$.

17) $T(2u) = 2T(u) = 2\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$

$T(3v) = 3T(v) = 3\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$.

$T(2u+3v) = T(2u) + T(3v) = 2T(u) + 3T(v) = 2\begin{bmatrix} 4 \\ 1 \end{bmatrix} + 3\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 8 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 9 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5 \\ 11 \end{bmatrix}}}$.

21b) No; domain $= \mathbb{R}^5$.

c) No. (Range = Image). Eg. A = zero matrix, then
(unless $m=0$)
image $= \{\underline{0}\}$.

e) True. Linear $\Rightarrow T(c_1\underline{v}_1 + c_2\underline{v}_2) = T(c_1\underline{v}_1) + T(c_2\underline{v}_2) = c_1T(\underline{v}_1) + c_2T(\underline{v}_2)$

For the converse, taking $c_1 = c_2 = 1$ gives one rule, & $c_2 = 0, \underline{v}_2 = \underline{0}$ the other

24) $T(\underline{x}+\underline{y}) = A(\underline{x}+\underline{y}) + \underline{b}$.

$T(\underline{x}) + T(\underline{y}) = A\underline{x} + \underline{b} + A\underline{y} + \underline{b} = A(\underline{x}+\underline{y}) + 2\underline{b}$.

So if T linear then $\underline{b} = 2\underline{b}$ so $\underline{0} = \underline{b}$.

32) ~~[illegible]~~ $c = -1$, $\underline{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(c\underline{v}) = T\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ $\neq$ so not linear.

$cT(\underline{v}) = -T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = -\begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

2) $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 9 & -8 \end{bmatrix}$

15) $A = \begin{bmatrix} 2 & -4 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 3 \end{bmatrix}$