Week 6 HW sol'ns lin alg 1.

1) $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}^{-1} = \frac{1}{32-30}\begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -5/2 & 4 \end{bmatrix}$

5) $A\underline{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ Unique sol'n $A^{-1}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$

8) $A = PBP^{-1} \leadsto P^{-1}A = BP^{-1} \leadsto \underline{P^{-1}AP = B}$.

16) Let $C = AB$, then $A = CB^{-1}$. Then let $D = BC^{-1}$,

so $DA = CB^{-1}BC^{-1} = I_n$ & $AD = CB^{-1}BC^{-1} = I_n$,

so D is an inverse to A.

25) Case 1: $a = b = 0$. then $\begin{bmatrix} 0 & 0 \\ c & d \end{bmatrix}\begin{bmatrix} \lambda d \\ -\lambda c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ for any $\lambda \in \mathbb{R}$,

so ∞ many sol'ns.

<u>Case 2</u>: Say $a \neq 0$ or $b \neq 0$. Then

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -b \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, & $\begin{bmatrix} -b \\ a \end{bmatrix}$ is a non-zero solution of a homogeneous system, so there are ∞ many solutions.

29) $\left[\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ 4 & -9 & 0 & 1 \end{array}\right] \xrightarrow{r_2 \mapsto r_2 - 4r_1} \left[\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ 0 & 3 & -4 & 1 \end{array}\right] \xrightarrow{r_1 \mapsto r_1 + r_2} \left[\begin{array}{cc|cc} 1 & 0 & -3 & 1 \\ 0 & 3 & -4 & 1 \end{array}\right]$

$\leadsto \left[\begin{array}{cc|cc} 1 & 0 & -3 & 1 \\ 0 & 1 & -4/3 & 1/3 \end{array}\right]$ $\underline{A^{-1} = \begin{bmatrix} -3 & 1 \\ -4/3 & 1/3 \end{bmatrix}}$

31) $$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}.$$

13) No, eg $[0]$.

11a) True. Only trivial sol'n $\Rightarrow$ no free variables $\Rightarrow$ pivot in every column $\Rightarrow$ n pivots $\Rightarrow$ pivot in every row & column $\Rightarrow$ row equiv. to I_n.

b) True. Colums span $\mathbb{R}^n$ $\Rightarrow$ pivot in every row $\Rightarrow$ pivot in every column $\Rightarrow$ colums lin. indep.

c) False, eg. $n = 1$, $A = [0]$, $\underline{b} = [1]$

d) True; nontrivial sol'n $\Rightarrow$ free variable $\Rightarrow$ at least one column w/o a pivot.

e) True; A^T not invertible $\Rightarrow$ $\det(A^T) = 0$ $\Rightarrow$ A not invertible.
$\det(A^T) = \det(A)$

1) $\det\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix} = 3\det\begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix} + 4\det\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

$= 3(-3-10) + 4(10-0)$

$= -39 + 40 = \underline{1}$

9) ~~det~~ 3rd row:

$\det A = 2\det\begin{bmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{bmatrix} = 10\det\begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix} = 10(7-6) = \underline{10}.$

5) $\det\begin{bmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{bmatrix} = \det\begin{bmatrix} 0 & 1 & -2 \\ -1 & -4 & 4 \\ 0 & 1 & 1 \end{bmatrix} = \det\begin{bmatrix} 1 & 4 & -4 \\ 0 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

$= \det\begin{bmatrix} 1 & 4 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix} = 3\times1\times1 = \underline{3}.$

7) $\det\begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{bmatrix} = \det\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & -4 & 2 & -5 \end{bmatrix} = \det\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$= \underline{0}.$

19) $\left|\det\begin{bmatrix} 5 & 6 \\ 2 & 4 \end{bmatrix}\right| = |20-12| = 8$

20) $\left|\det\begin{bmatrix} -1 & 4 \\ 3 & -5 \end{bmatrix}\right| = |5-12| = 7.$