Week 6 Homework outline Solutions

1) $\det\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix} = 3\det\begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix} + 4\det\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

$= 3(-3-10) + 4(10-0)$

$= -39 + 40 = \underline{1}$

9) ~~det~~ 3rd row:

$\det A = 2\det\begin{bmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{bmatrix} = 10\det\begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix} = 10(7-6) = \underline{10}.$

5) $\det\begin{bmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{bmatrix} = \det\begin{bmatrix} 0 & 1 & -2 \\ -1 & -4 & 4 \\ 0 & 1 & 1 \end{bmatrix} = \det\begin{bmatrix} 1 & 4 & -4 \\ 0 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

$= \det\begin{bmatrix} 1 & 4 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix} = 3\times1\times1 = \underline{3}.$

7) $\det\begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{bmatrix} = \det\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & -4 & 2 & -5 \end{bmatrix} = \det\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

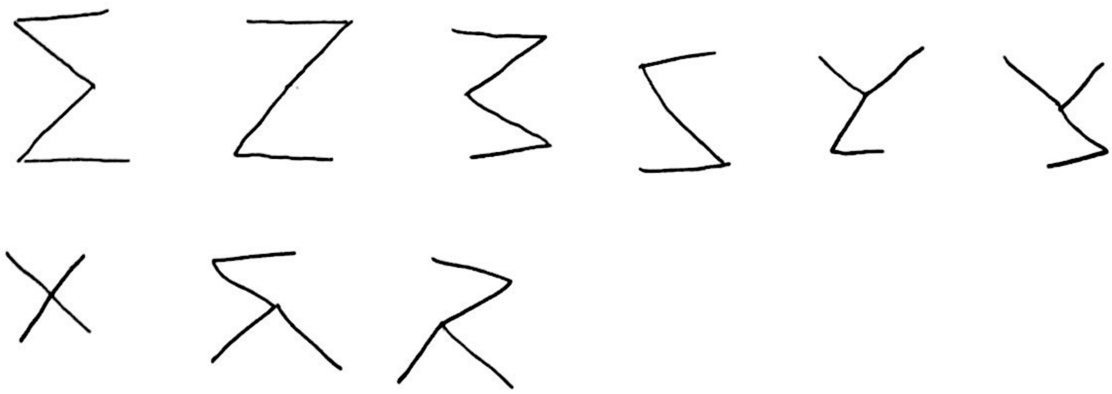
$= \underline{0}.$

19) $\left|\det\begin{bmatrix} 5 & 6 \\ 2 & 4 \end{bmatrix}\right| = |20-12| = 8$

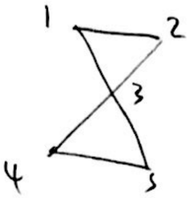
20) $\left|\det\begin{bmatrix} -1 & 4 \\ 3 & -5 \end{bmatrix}\right| = |5-12| = 7$

Spanning Trees

1)



2)



Laplacian depends on ordering.

$$L = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix} \end{array}$$

$$\operatorname{cof}_{11} L = (-1)^2 \det \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 4 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} = \det \begin{bmatrix} 0 & 7 & -2 & -2 \\ -1 & 4 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$= \det \begin{bmatrix} 7 & -2 & -2 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} = \det \begin{bmatrix} 7 & -2 & -2 \\ 0 & 3 & -3 \\ -1 & -1 & 2 \end{bmatrix}$$

$$= 3 \det \begin{bmatrix} 7 & -2 & -2 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix} = 3 \left[7 \cdot (2-1) + (-1)(2+2) \right]$$

$$= 3 \cdot 3 = 9$$

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7. $A = \begin{bmatrix} 6s & 4 \\ 9 & 2s \end{bmatrix}$ $\quad \underline{b} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

$\det A = 12s^2 - 36,$
$= 12(s^2-3)$ so a unique solution exists unless $s = \pm\sqrt{3}$.

Assuming $\det A \neq 0$:

$A_1(\underline{b}) = \begin{bmatrix} 5 & 4 \\ -2 & 2s \end{bmatrix}$, $\det(A_1 \underline{b}) = 10s + 8$

$$x_1 = \frac{10s+8}{12(s^2-3)} = \frac{5s+4}{6(s^2-3)}$$

$A_2(\underline{b}) = \begin{bmatrix} 6s & 5 \\ 9 & -2 \end{bmatrix}$, $\det(A_2(\underline{b})) = -12s - 45$

$$x_2 = \frac{-12s-\cancel{3}45}{12(s^2-3)} = \frac{4s+15}{4(3-s^2)},$$

~~gen. so~~ solution

$$\underline{x} = \begin{bmatrix} \dfrac{5s+4}{6(s^2-3)} \\[2ex] \dfrac{4s+15}{4(3-s^2)} \end{bmatrix}.$$

8. (t in place of s for clarity):

$A = \begin{bmatrix} 3t & -5 \\ 9 & 5t \end{bmatrix}$, $\underline{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$,

$\det A = 15t^2 + 45 = 15(t^2+3)$, so unique solution for all $t \in \mathbb{R}$.

$A_1(\underline{b}) = \begin{bmatrix} 3 & -5 \\ 2 & 5t \end{bmatrix}$, $\det(A_1 \underline{b}) = 15t + 10$

$A_2(\underline{b}) = \begin{bmatrix} 3t & 3 \\ 9 & 2 \end{bmatrix}$, $\det(A_2 \underline{b}) = 6t - 27$

Solution

$$\underline{x} = \begin{bmatrix} \dfrac{3t+2}{3(t^2+3)} \\[2ex] \dfrac{2t-9}{5(t^2+3)} \end{bmatrix}$$