Read the section `Subspaces of a finite dimensional space' - it will make these questions much easier. We will apply the `Basis theorem' several time.

Determine whether the sets in Exercises 1–8 are bases for $\mathbb{R}^3$. Of the sets that are *not* bases, determine which ones are linearly independent and which ones span $\mathbb{R}^3$. Justify your answers.

1. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$

4. $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 4 \end{bmatrix}$

23. Suppose $\mathbb{R}^4 = \text{Span}\{v_1, \ldots, v_4\}$. Explain why $\{v_1, \ldots, v_4\}$ is a basis for $\mathbb{R}^4$.

14. The set $\mathcal{B} = \{1 - t^2, t - t^2, 2 - t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $p(t) = 1 + 3t - 6t^2$ relative to $\mathcal{B}$.

In Exercises 29 and 30, V is a nonzero finite-dimensional vector space, and the vectors listed belong to V. Mark each statement True or False. Justify each answer. (These questions are more difficult than those in Exercises 19 and 20.)

29. a. If there exists a set $\{v_1, \ldots, v_p\}$ that spans V, then $\dim V \le p$.

 b. If there exists a linearly independent set $\{v_1, \ldots, v_p\}$ in V, then $\dim V \ge p$.

 c. If $\dim V = p$, then there exists a spanning set of $p + 1$ vectors in V.

In Exercises 1–4, assume that the matrix A is row equivalent to B. Without calculations, list ~~rank A and~~ $\dim \operatorname{Nul} A$. Then find bases for $\operatorname{Col} A$, ~~Row A,~~ and $\operatorname{Nul} A$.

1. $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$