In Exercises 1–4, find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the given data points.

1. $(0, 1), (1, 1), (2, 2), (3, 2)$

2. $(1, 0), (2, 1), (4, 2), (5, 3)$

5. Let X be the design matrix used to find the least-squares line to fit data $(x_1, y_1), \dots, (x_n, y_n)$. Use a theorem in Section 6.5 to show that the normal equations have a unique solution if and only if the data include at least two data points with different x-coordinates.

7. A certain experiment produces the data $(1, 1.8)$, $(2, 2.7)$, $(3, 3.4)$, $(4, 3.8)$, $(5, 3.9)$. Describe the model that produces a least-squares fit of these points by a function of the form

$$y = \beta_1 x + \beta_2 x^2$$

Such a function might arise, for example, as the revenue from the sale of x units of a product, when the amount offered for sale affects the price to be set for the product.

Write a matrix equation Av = b such that the least-squares solution v = [\beta_1, \beta_2] gives the best-fitting function
y = \beta_1 x + \beta_2 x^2.