Solutions LA2 week 3.

1) $\det\left(\begin{bmatrix}3 & 2\\ 3 & 8\end{bmatrix} - 2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right) = \det\begin{pmatrix}1 & 2\\ 3 & 6\end{pmatrix} = 0$

so IS an eigenvalue.

2) $\det\left(\begin{pmatrix}-1 & 4\\ 6 & 9\end{pmatrix} + 3\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right) = \det\begin{pmatrix}2 & 4\\ 6 & 12\end{pmatrix} = 24 - 24 = 0$ ~~≠0~~

so IS an eigenvalue.

3) $\begin{bmatrix}1 & -1\\ 6 & -4\end{bmatrix}\begin{bmatrix}1\\ 3\end{bmatrix} = \begin{bmatrix}-2\\ -6\end{bmatrix} = -2\begin{bmatrix}1\\ 3\end{bmatrix}$

so is an eigenvector, with eigenvalue -2.

19.) $\lambda = 1$: $\begin{bmatrix}3 & 0\\ 2 & 1\end{bmatrix} - 1\cdot\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}2 & 0\\ 2 & 0\end{bmatrix} \xrightarrow{\text{R.Red}} \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$

W_1 has basis $\left\{\begin{bmatrix}0\\ 1\end{bmatrix}\right\}$

$\lambda = 3$: $\begin{bmatrix}3 & 0\\ 2 & 1\end{bmatrix} - \begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} = \begin{bmatrix}0 & 0\\ 2 & -2\end{bmatrix} \xrightarrow{\text{R.Red}} \begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix}$,

W_3 has basis $\left\{\begin{bmatrix}1\\ 1\end{bmatrix}\right\}$.

17.) $\det\begin{pmatrix}-\lambda & 0 & 0\\ 0 & 3-\lambda & 4\\ 0 & 0 & -2-\lambda\end{pmatrix} = (-\lambda)\left[(3-\lambda)(-2-\lambda)\right] = -\lambda(\lambda+2)(\lambda-3)$

eigenvalues 0, -2, 3.

21 a) False, $\underline{x}$ could $= \underline{0}$.

b) True, A not invertible $\Leftrightarrow \det A = 0$

$\Leftrightarrow \det(A - 0I_n) = 0$

$\Leftrightarrow$ 0 an eigenvalue.

c) True, by definition

d) True, just check whether $A\underline{v}$ is a scalar multiple of $A\underline{v}$

e) False (it doesn't help).

24) eg $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

25) Say $A\underline{x} = \lambda \underline{x}$ some $\underline{x} \neq \underline{0}$. Then $A^{-1}A\underline{x} = A^{-1}\lambda\underline{x}$

$$\underbrace{A^{-1}A\underline{x}}_{= \underline{x}} = \underbrace{A^{-1}\lambda\underline{x}}_{= \lambda A^{-1}\underline{x}}$$

so $A^{-1}\underline{x} = \lambda^{-1}\underline{x}$.

26) Say A has a non-zero e-value λ w. e-vector $\underline{v}$. Then

$\underline{0} = A^2\underline{v} = A(A\underline{v}) = A(\lambda\underline{v}) = \lambda(A\underline{v}) = \lambda^2\underline{v}$, ~~a contradiction~~ which is impossible.

27) λ e-value of $A \Leftrightarrow \det(A - \lambda I_n) = 0$,

but $\det(A - \lambda I_n) = \det((A - \lambda I_n)^T) = \det(A^T - \lambda I_n)$,

so $\det(A - \lambda I_n) = 0 \Leftrightarrow \det(A^T - \lambda I_n) = 0$

$\Leftrightarrow \lambda$ is an e-value of A^T.