Retake Exam – Lineaire Algebra 2

11 July 2017

Time: 3 hours.

Fill in your name and student number on all papers you hand in.

In total there are 6 question, and each question is worth the same number of points.

In all questions, justify your answer fully and show all your work.

In this examination you are only allowed to use a pen and examination paper.

Question 1

Let A be the matrix

$$A = \begin{bmatrix} 4 & 0 & 2 \\ 3 & 1 & 2 \\ -15 & 3 & -6 \end{bmatrix}.$$

a) Find the eigenvalues of A.

b) For each eigenvalue in (a) give a basis for the corresponding eigenspace.

Question 2

Consider the following set S of vectors in $\mathbb{R}^3$:

$$S = \{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} \}.$$

a) Is the set S linearly independent?

b) Does the set S span $\mathbb{R}^3$?

c) What is the dimension of the span of S?

Question 3

Consider the vectors

$$\mathbf{u} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

a) Are the vectors $\mathbf{u}$ and $\mathbf{v}$ orthogonal?

b) What is the distance between $\mathbf{u}$ and $\mathbf{v}$?

c) Find a vector $\mathbf{w} \in \mathbb{R}^3$ such that $\mathbf{w}$ has length $\sqrt{42}$ and is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

Question 4

Let A be the matrix

$$A = \begin{bmatrix} 17 & -12 \\ 24 & -17 \end{bmatrix}.$$

a) Show that A has eigenvalues 1 and -1.

b) For each eigenvalue in (a) give a basis for the corresponding eigenspace.

c) Give an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

d) Calculate A^{100}.

Question 5

Define a matrix A by

$$A = \begin{bmatrix} 0 & 2 & -5 \\ -5 & 2 & 3 \\ 5 & -2 & -5 \end{bmatrix}.$$

a) Use the Gram-Schmidt algorithm to turn the columns of A into an orthogonal set.

b) Are the columns of A linearly independent?

c) Are the rows of A linearly independent?

d) What is the rank of A?

Question 6

For each of the following 5 statements, say whether the statement is true or false. Justify your answer (either by an example if the statement is false, or a brief proof if it is true). If you are unsure, try some small examples.

a) There are only finitely many linear maps from a 1-dimensional vector space to itself.

b) If U and V are linear subspaces of $\mathbb{R}^n$, then the intersection $U \cap V$ is a linear subspace of $\mathbb{R}^n$.

c) If A and B are invertible $n \times n$ matrices, then AB is also invertible.

d) If A and B are invertible $n \times n$ matrices, then $A + B$ is also invertible.

e) There is exactly one vector in $\mathbb{R}^1$ of length 5.