Retake Exam – Lineaire Algebra 1

17 March 2017

Time: 3 hours.

Fill in your name and student number on all papers you hand in.

In total there are 6 questions, and each question is worth the same number of points.

In all questions, justify your answer fully and show all your work.

In this examination you are only allowed to use a pen and examination paper.

1. Consider the system of linear equations:

$$\begin{aligned} -4x_1 + 4x_3 + 2x_4 &= 4 \\ 2x_2 + 2x_3 - 3x_4 &= 4 \\ 2x_1 - 2x_3 &= 0 \end{aligned}$$

   a) Write down the augmented coefficient matrix of the linear system above.
   b) Row reduce the augmented coefficient matrix that you wrote down.
   c) Write down 3 distinct solutions of the linear system.

2. Define a matrix A and vectors $\underline{b}$, $\underline{c}$ by

$$A = \begin{bmatrix} 0 & 5 & 1 \\ 0 & -4 & -1 \\ -1 & -5 & 3 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1 \\ 2 \\ -14 \end{bmatrix}, \quad \underline{c} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

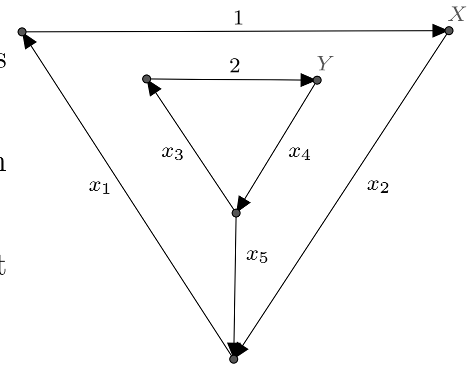
   a) Compute the determinant of A.
   b) How many solutions does the linear system $A\underline{x} = \underline{b}$ have?
   c) Compute the inverse of A.
   d) Write down a solution of the linear system $A\underline{x} = \underline{c}$.

3. Consider the vectors in $\mathbb{R}^3$ given by

$$\underline{a} = \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}, \quad \underline{c} = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}.$$

   a) Is the set $\{\underline{a}, \underline{b}\}$ linearly independent?
   b) Is the vector $\underline{c}$ contained in the span of $\{\underline{a}, \underline{b}\}$?
   c) Does the set $\{\underline{a}, \underline{b}, \underline{c}\}$ span $\mathbb{R}^3$?

4. Consider the network on the right.



   a) Write down a linear system describing the flow in this network.

   b) Put the augmented matrix of the linear system from (a) in row reduced echelon form.

   c) Suppose we also drew an edge from X to Y. Would it still hold that the flow along edge x_5 must be zero?

5. For each of the following 5 statements, say whether the statement is true or false. Justify your answer (either by an example if the statement is false, or a brief proof if it is true). If you are unsure, try some small examples.

   a) There exists an injective linear map from $\mathbb{R}^2$ to $\mathbb{R}^3$.

   b) If a linear system has zero as a solution then it must be homogeneous

   c) For every two $n \times n$ matrices A and B we have $A^T + B^T = (A + B)^T$.

   d) If G is a connected graph with n vertices and T is a spanning tree of G, then T has exactly n edges.

   e) Let A be an $n \times n$ matrix, and let B be the matrix obtained from A by adding a copy of the first row to every row. Then $\det B = 2 \det A$.

6. Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

   a) What is $E_1 A$?

   b) What is AE_1?

   c) What is $E_2 A$?

   d) What is AE_2?

   e) Write down a 3×3 matrix E_3 such that

$$E_3 A = \begin{bmatrix} 1 & 2 & 3 \\ 8 & 10 & 12 \\ 7 & 8 & 9 \end{bmatrix}.$$