Universiteit Leiden
Wiskunde en Natuurwetenschappen

LA1, 13 Jan 2017.

Vak: ______________________ Naam: ______________________

Datum: ______________________ Studierichting: ______________________

Docent: ______________________ Collegekaartnummer: ______________________

1a) $\left[\begin{array}{cccc|c} 2 & 3 & 3 & -3 & 2 \\ -4 & -4 & -2 & 2 & 0 \\ 3 & 0 & -3 & 3 & 0 \end{array}\right]$.

b) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & -1 & 4 \end{array}\right]$.

c) $x_1 = 4$
$x_2 = -6$
$x_3 = x_4 + 4$
x_4 is free.

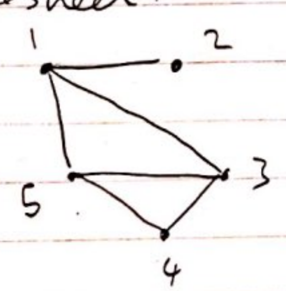
eg. $x_4 = 0$: $\begin{bmatrix} 4 \\ -6 \\ 4 \\ 0 \end{bmatrix}$ $x_4 = 1$: $\begin{bmatrix} 4 \\ -6 \\ 5 \\ 1 \end{bmatrix}$.

2a) $\det A = 1$

b) $A^{-1} = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 6 & 29 \\ 0 & -1 & -5 \end{bmatrix}$

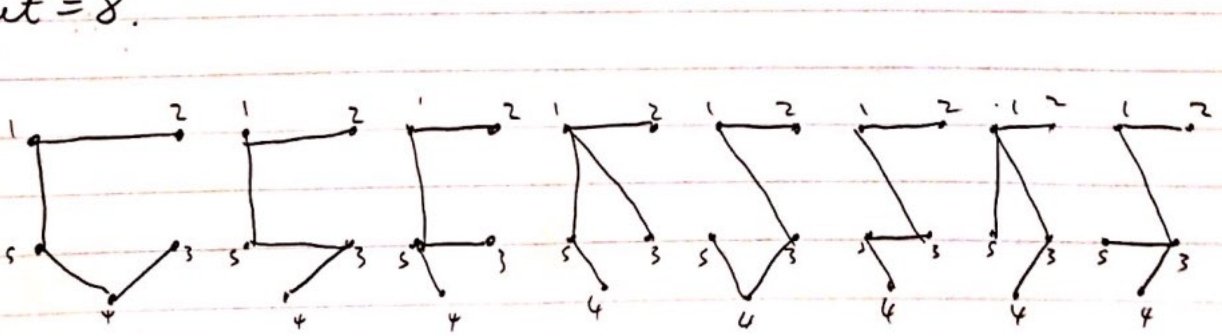
c) $\det(A^{-1}) = \frac{1}{\det A} = 1$

3) See sheet.

4)
a)


b) $\det = 8$.

c) 8

d)


Scale does not matter, because the functions are linear.
So choose a convenient scale for each question.

EXTRA SHEET WITH QUESTION 3 OF THE LINEAR ALGEBRA 1 EXAM
JANUARY 2017

Name:
Student number:

$A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$

$A\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$A\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

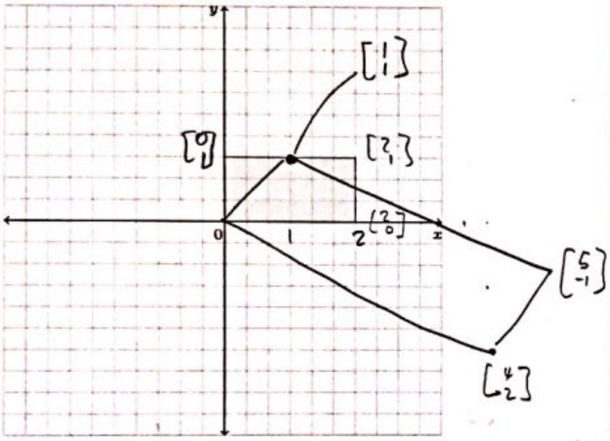
$A\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

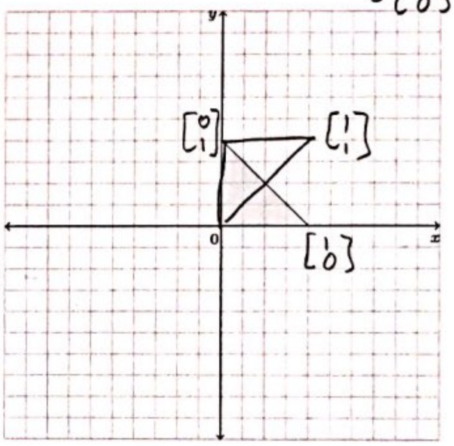
$B\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$B\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

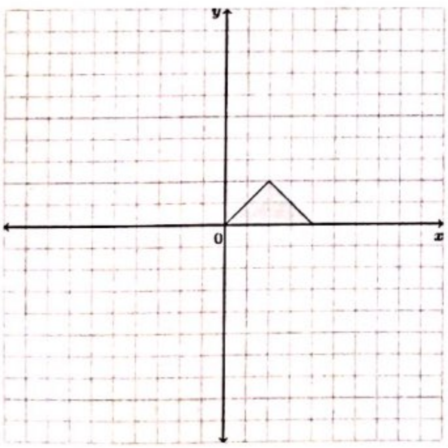
$B\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.



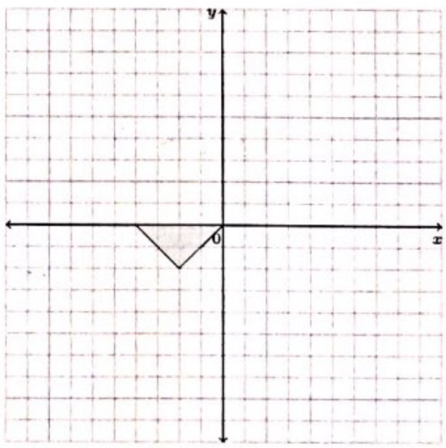
(a)



(b)



(c)



(d)

Matrix sending figure (c) to figure (d) (you can write on the back if you need more space): $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

or $\begin{bmatrix} -1/2 & -3/2 \\ -1/2 & 1/2 \end{bmatrix}$.

5) a) False, e.g. A = zero matrix.

b) False, eg $A = B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. $A^{-1} = B^{-1} = A$, so

$$A^{-1} + B^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

$$A + B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \text{so } (A+B)^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}.$$

c) True. The row corresponds to an equation

$$0x_1 + 0x_2 + \dots + 0x_n = a \quad \text{with } a \neq 0,$$

which simplifies to $0 = a$, which is clearly impossible.

d) False. $\det B = 2^n \det A$. Eg $n=3$, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

$\det A = 1$, $\det B = 8$.

e) True. Say $r_n = r_{n-1}$. By the ERO $r_n \mapsto r_n - r_{n-1}$, we arange a row of zeros. Expanding along that row gives determinant 0.

6a) $2\underline{v} - \underline{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

b) $T(2\underline{v} - \underline{u}) = 2T(\underline{v}) - T(\underline{u}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

c) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{u} - \frac{3}{2}\underline{v}$. $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T(\underline{u} - \frac{3}{2}\underline{v}) = T(\underline{u}) - \frac{3}{2}T(\underline{v}) = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

e) Standard matrix is $\left(T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \; T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \right) = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$.