Retake Exam – Lineaire Algebra 1

Date

Time: 3 hours.

Solutions

1. a)

5 $$\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5 b) The solution set is the span of the vectors $\begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ *and* $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

3 2. a) $A^{-1} = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -5 & -4 \\ 0 & -1 & -1 \end{bmatrix}$

3 b) $\det B = 6$

3 c) $AB = \begin{bmatrix} -8 & 9 & -4 \\ 2 & 1 & 0 \\ -5 & 0 & -1 \end{bmatrix}$

1 d) $\det(-B) = -6$

3. 1 a) $-3\underline{u} + 1\underline{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

2 b) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

4 c) Observe that $2\underline{u} + -1\underline{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, hence $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$.

3 d) The standard matrix for T is $\begin{bmatrix} -2 & 5 \\ 4 & -1 \end{bmatrix}$.

4. 2 a) Linear; it comes by matrix multiplication.

2 b) Linear (direct from the axioms)

2 c) Not linear; $\begin{bmatrix} -1 \\ -1 \end{bmatrix} = -f_c(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) \neq f_c(\begin{bmatrix} -1 \\ -1 \end{bmatrix}) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

2 d) Not linear; $\begin{bmatrix} -3 \\ -5 \end{bmatrix} = -f_a(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = -f_a \circ f_c(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) \neq f_a \circ f_c(\begin{bmatrix} -1 \\ -1 \end{bmatrix}) = f_a(\begin{bmatrix} -1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

2 2 e) Linear, because it sends every vector to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, i.e. it equals f_b.

4 5. a) Linear system with matrix (free to permute rows!)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & -1 & 0 & -4 \\ 1 & -1 & 0 & 0 & 0 & 4 \end{bmatrix}$$

5 b) Reduced echelon form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1 c) Yes, there does exist a solution such that the flow along the edge labelled x_3 is greater than 100, for example

$$\begin{bmatrix} 1 \\ -3 \\ 101 \\ 105 \\ 101 \end{bmatrix}.$$

2 6. a) False, e.g. $A = -B$ with A invertible.

2 2 b) False, e.g. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

2 2 c) True; by a row replacement (which does not change the determinant) we can arrange for A to have a zero row, then cofactor expand along that row.

d) True; write the system as $Ax = b$, then if 0 is a solution we see that $b = 0$.

e) True; $\det A^2 = (\det A)^2$, so this is non-zero if and only if $\det A$ is non-zero.