Universiteit Leiden
Wiskunde en Natuurwetenschappen

Vak: LA1 | Naam:
Datum: Jan 2018 | Studierichting:
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Q1 a)

$$\begin{bmatrix} -1 & 2 & 2 & -4 \\ 1 & 2 & 2 & 0 \\ 4 & 1 & 2 & -1 \end{bmatrix} \xrightarrow{r_2 \mapsto r_2 - r_1} \begin{bmatrix} -1 & 2 & 2 & -4 \\ 2 & 0 & 0 & 4 \\ 4 & 1 & 2 & -1 \end{bmatrix} \xrightarrow[r_2 \mapsto \frac{1}{2} r_2]{r_1 \mapsto r_1 + r_2} \begin{bmatrix} 1 & 2 & 2 & 0 \\ 1 & 0 & 0 & 2 \\ 4 & 1 & 2 & -1 \end{bmatrix}$$

$r_1 \leftrightarrow r_2$
$r_3 \mapsto r_3 - 4r_1$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 2 & 2 & 0 \\ 0 & 1 & 2 & -9 \end{bmatrix} \xrightarrow{r_2 \mapsto r_2 - r_1} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 2 & -2 \\ 0 & 1 & 2 & -9 \end{bmatrix} \xrightarrow[r_3 \mapsto r_3 - r_2]{r_2 \mapsto \frac{1}{2} r_2} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -8 \end{bmatrix}$$

$r_2 \mapsto r_2 - r_3$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -8 \end{bmatrix}$$

b) Yes. A has a pivot in every row, so colums span all of $\mathbb{R}^3$.

c)
$$\begin{aligned} x_1 + 2x_4 &= 0 \\ x_2 + 7x_4 &= 0 \\ x_3 - 8x_4 &= 0 \\ x_4 - 1x_4 &= 0 \end{aligned}$$

General solution $= \operatorname{Span}\left\{ \begin{bmatrix} -2 \\ -7 \\ 8 \\ 1 \end{bmatrix} \right\}$.

Q2 a) $\det A = 4$

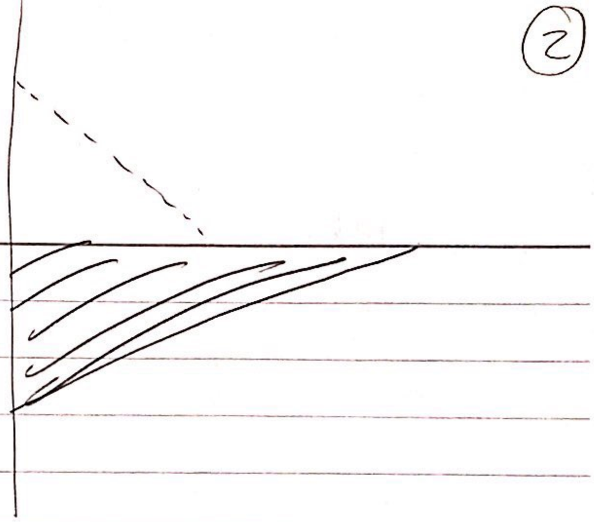
b) $B^{-1} = \begin{bmatrix} 1 & 2 & 6 \\ 0 & 0 & 1 \\ -1 & -3 & -8 \end{bmatrix}$

d) $\det(A^2) = (\det(A))^2 = 16$

c) $AB = \begin{bmatrix} -1 & 7 & 0 \\ 3 & -1 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

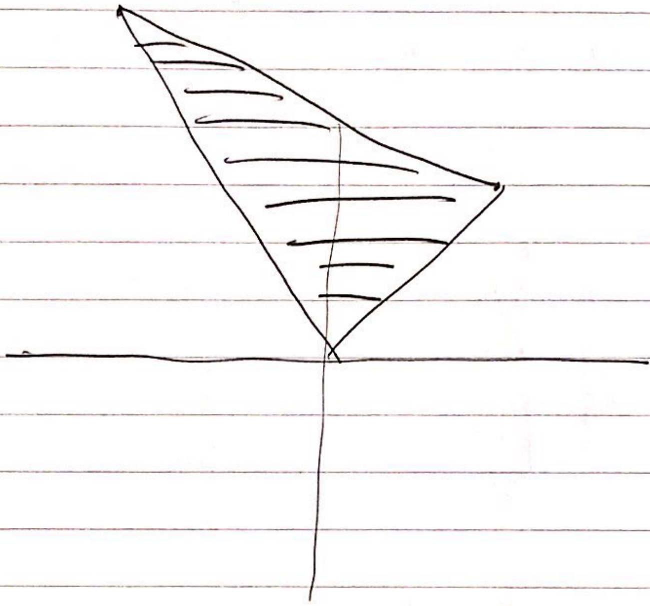
3a) $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$



b) $\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

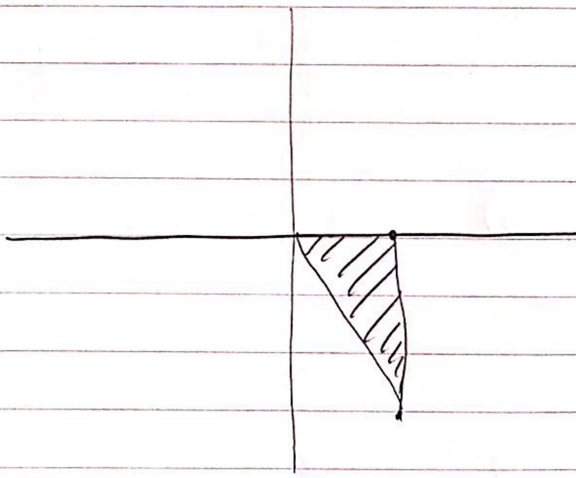
$\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$



c) $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & -1 \end{bmatrix}$

$A^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$

$A^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -1 \end{bmatrix}$



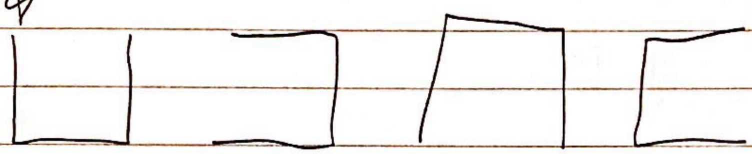
d) $\cancel{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}$ $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

4) a)
$$L = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} \end{array}$$

b) $$\operatorname{cof}_{2,2} L = (-1)^4 \det \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} = 4$$

c) 4

d)



5a) $$A = \begin{bmatrix} 1 & -1 & 0 \\ 3 & 0 & -1 \\ 4 & -1 & -1 \end{bmatrix}$$

b) $\det A = 0$, so no.

c) $\det A = 0$, so no.

d) eg $\underline{v} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, then we row-reduce $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & -1 & -1 & 1 \end{bmatrix}$

$$\xrightarrow{r_3 \mapsto r_3 - r_1 - r_2} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_2 \mapsto r_2 - 3r_1} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So have pivot in every row, so colums span $\mathbb{R}^3$.

6a) True; Say $r_i = r_j$, & let B obtained from A by $r_i \to r_i - r_j$,

then $\det B = \det A$, & row i of B is 0,

so $\det B = 0$ by cofactor expansion along row i.

b) False, eg. $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\underline{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

c) True. $\left(\frac{1}{3} A^{-1}\right)(3A) = \frac{3}{3} AA^{-1} = I_4$

d) False, eg. $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, all $\underline{x}$ are solutions, no pivots.

e) False, eg. $A = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{\text{row op}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$,

$\operatorname{Span}\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\} \neq \operatorname{Span}\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$